Chapitre 7.  Lateral Earth Pressures

7.1 Introduction

The pressure at any point in a fluid such as water is the same in all directions. Thus the lateral pressure on a vertical surface retaining water is equal to \( \gamma_w h \) where \( h \) = the height of water above the point considered. Fig. 1 shows the lateral pressure diagram on a wall of height \( H \) retaining water.

The total force \( P \) per unit length of wall will be equal to the area of the pressure diagram.

\[
P = \frac{1}{2} \gamma_w H^2
\]

and this force will act at the centroid of the diagram, i.e. at \( 2H/3 \) from the surface.

Figure 1: lateral pressure in water  
Figure 2: lateral pressure in soil
In the case of soil, which, unlike water, possesses resistance to shearing, the lateral pressure at any point will not be the same as the vertical pressure at that point (Fig. 2).

In a homogeneous natural soil deposit, the ratio \( \sigma_h'/\sigma_v' \) is a constant known as coefficient of earth pressure at rest (\( K_0 \)).

For normally consolidated clays and granular soils, \( K_0 \approx 1 - \sin \phi' \)

In order to design soil-retaining structures such as retaining walls\(^2\) and sheet pile walls\(^3\), it is necessary to determine the magnitude of the lateral pressures to which the structure is subjected.

The lateral pressure behind a wall will vary depending on whether the wall is going away from soil or towards soil.

### 7.2 Lateral Earth Pressures In Granular Soils – Rankine theory

**Hypothesis**: the ground surface is horizontal and there is no friction between the wall and the soil.

Let's imagine a sheet pile wall (Fig. 3.) and let's look at the soil elements A and B during the wall movement caused by the earth pressures at the right of the wall.

#### 7.2.1 Point A : Active soil pressure

In A, the earth pressure is called "active", because the soil in A is responsible of the wall movement.

![Figure 3 : Active soil pressure](image)

Initially, there is no lateral movement, thus, at this time, the Mohr circle (Fig. 4) has two principal stresses

\[
\sigma_v' = \gamma z \quad \text{and} \quad \sigma_h' = K_0 \sigma_v' = K_0 \gamma z
\]

As the wall moves away from the soil,

- \( \sigma_v' \) remains the same and

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\(^2\) Murs de soutènement  
\(^3\) rideau de palplanches
\[ \sigma_h' \] decreases till failure occurs.

The Mohr circle changes thus during the movement and at failure, it is tangent to the Mohr-Coulomb failure line (Fig.4).

\[ \sigma_v' = K_A \sigma_v' \]

*Where* \( K_A \) is called the Rankine’s coefficient of active earth pressure.
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\[ K_A = \frac{\sigma_{h,active}'}{\sigma_v'} = \frac{x - r}{x + r} = \frac{1 - \frac{r}{x}}{1 + \frac{r}{x}} = \frac{1 - \sin \phi'}{1 + \sin \phi'} = \tan^2 \left( 45 - \phi' / 2 \right) \]

The displacement of the wall necessary to reach the active Rankine state is about \( H/1000 \), where \( H \) is the height of the wall.

**7.2.2 Point B : Passive soil pressure**

Now, let’s look at the soil element B during the wall movement caused by the earth pressures at the right of the wall.

The soil in B resists to the wall movement, thus the earth pressure in B is called "passive".

Initially, there is no lateral movement, thus the Mohr circle (Fig.6) has two principal stresses

\[ \sigma_v' = \gamma z \quad \text{and} \quad \sigma_h' = K_0 \sigma_v' = K_0 \gamma z \]

As the wall moves away from the soil,

- \( \sigma_v' \) remains the same and
- \( \sigma_h' \) increases till failure occurs.

At failure, the Mohr circle is tangent to the Mohr-Coulomb failure line (Fig.6).

![Figure 6: Mohr circle under Passive Soil Pressure](image)

As we have seen in chapter 6 (exercise 1), the failure plane is at \( 45^\circ - \phi / 2 \) to horizontal (Fig. 7).
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**Figure 7 : Mohr circle at Rankine Passive Limit State**

$$
\sigma'_{h,\text{passive}} = K_p \sigma'_{v}
$$

Where $K_p$ is called the Rankine’s coefficient of passive earth pressure

$$
K_p = \frac{\sigma'_{h,\text{passive}}}{\sigma'_{v}} = \frac{x + \frac{r}{x}}{x - \frac{r}{x}} = \frac{1 + \sin \phi'}{1 - \sin \phi'} = \tan^2(45 + \phi'/2)
$$

The displacement of the wall necessary to reach the Rankine passive state is about $H/100$, where $H$ is the height of the wall. The necessary movement of the wall is thus 10 times greater than the one necessary to reach the active state.

**7.2.3 Summary**

In the active state, $\sigma'_1 = \sigma'_{v}$ and $\sigma'_3 = \sigma'_h$.
In the passive state, $\sigma'_1 = \sigma'_h$ and $\sigma'_3 = \sigma'_{v}$.
7.3 Lateral Earth Pressures In Cohesive Soils – Rankine theory

By similar developments, we find, in case of cohesive soils:

$$\sigma'_1 = \sigma'_3 \tan^2(45 + \phi'/2) + 2c' \tan(45 + \phi'/2)$$

$$\sigma'_3 = \sigma'_1 \tan^2(45 - \phi'/2) - 2c' \tan(45 - \phi'/2)$$

In the active state, $\sigma'_1 = \sigma'_v$ and $\sigma'_3 = \sigma'_h$.

In the passive state, $\sigma'_1 = \sigma'_h$ and $\sigma'_3 = \sigma'_v$.

In practice, for long term calculations, the cohesion is not taken into account in the calculations, because it is impossible to be sure that it will be always present. It is thus safer to neglect it.

7.4 Exercise

A retaining vertical wall 6 m high supports cohesionless dry soil of dry unit weight 16.3 kN/m³, effective angle of friction $35^\circ$ and void ratio 0.68. The surface of the soil is horizontal and level with the top of the wall. Neglecting wall friction, determine the total active earth thrust per metre of wall and at what height above the base of the wall the thrust acts.

7.5 Lateral Earth Pressure In Presence Of Groundwater

If due to poor drainage, the water table is 2.5 m below the ground surface in the previous exercise, determine the resulting lateral thrust on the wall and at what height above the base of the wall the thrust acts. Note: when the soil is saturated, the friction angle is $30^\circ$. 

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4 de niveau
5 poussée
6 nappe d'eau souterraine
**Step one:** Calculation of Saturated Unit Weights. (Assume a volume of solid of 1m³)

<table>
<thead>
<tr>
<th></th>
<th>Distribution by Volume (m³)</th>
<th>Distribution by Weight for the dry soil (kN)</th>
<th>Distribution by Weight for the saturated soil (kN)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Voids</td>
<td>V_v = e.V_s = e = 0.68</td>
<td>0</td>
<td>W_v = V_v - γ_v = 6.8</td>
</tr>
<tr>
<td>Solid</td>
<td>1</td>
<td>W_s</td>
<td>W_s</td>
</tr>
<tr>
<td><strong>TOTAL</strong></td>
<td>1.68</td>
<td><strong>W_s</strong></td>
<td><strong>W_s + 6.8</strong></td>
</tr>
</tbody>
</table>

\[ γ_{dry} = \frac{W_s \ kN}{1.68 \ m^3} = 16.3 \ kN / m^3 \]

**Hence** \( W_s = 27.384 \ kN \)

**and** \( γ_{sat} = \frac{W_s + 6.8 kN}{1.68 m^3} = 20.35 \ kN / m^3 \)

**Step two: calculations of the pressures on the wall**

The calculations are done, taking into account that:

- The earth pressures on the wall are calculated, under the water table, from the effective stresses.
- The water pressure is added because it is present between the soil particles and acts on the wall independently of the earth pressure.

**Figure 7 : Lateral pressures on the wall**

- Below the water table the angle of friction changes, so does \( K_a \). This explains the step at 2.5m depth in the active pressure diagram because the horizontal effective stress is different in the dry soil and in the wet soil at that depth.
Soil Mechanics: Lateral Earth Pressures

**DATA**

<table>
<thead>
<tr>
<th>Layer</th>
<th>Depth (m)</th>
<th>Density (kN/m³)</th>
<th>Angle (°)</th>
<th>K_a</th>
<th>( K_a = \tan^2 \left( 45 - \phi / 2 \right) )</th>
</tr>
</thead>
<tbody>
<tr>
<td>H_1</td>
<td>2.5</td>
<td>16.3</td>
<td>35</td>
<td>0.27099005</td>
<td></td>
</tr>
<tr>
<td>H_2</td>
<td>3.5</td>
<td>20.35</td>
<td>30</td>
<td>0.33333333</td>
<td></td>
</tr>
<tr>
<td>H_w</td>
<td>3.5</td>
<td>0</td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

**Active Pressures**

<table>
<thead>
<tr>
<th>Depth (m)</th>
<th>Active (kN/m²)</th>
<th>Total (kN/m²)</th>
<th>Water Pressure (kN/m²)</th>
<th>Forces</th>
</tr>
</thead>
<tbody>
<tr>
<td>DRY</td>
<td>0.0</td>
<td>0.0</td>
<td>0.0</td>
<td>0.0</td>
</tr>
<tr>
<td>2.5</td>
<td>40.8</td>
<td>40.8</td>
<td>0.0</td>
<td>13.8</td>
</tr>
<tr>
<td>WET</td>
<td>0.0</td>
<td>40.8</td>
<td>0.0</td>
<td>40.8</td>
</tr>
<tr>
<td>6.0</td>
<td>71.2</td>
<td>112.0</td>
<td>35.0</td>
<td>77.0</td>
</tr>
</tbody>
</table>

**Water pressures**

<table>
<thead>
<tr>
<th>Depth (m)</th>
<th>Active Side (kN/m²)</th>
<th>Water Pressure (kN/m²)</th>
<th>Forces</th>
</tr>
</thead>
<tbody>
<tr>
<td>DRY</td>
<td>2.5</td>
<td>0</td>
<td></td>
</tr>
<tr>
<td>WET</td>
<td>6.0</td>
<td>3.5</td>
<td>35.0</td>
</tr>
</tbody>
</table>

**TOTAL Active Side**

143.7 kN/m² | 239.1 kNm

\( x_{toe} (m) = 1.66374632 \)
7.6 Lateral Earth Pressures in case of inclined ground surface or friction at wall-ground interface

By now, we have considered the wall as perfectly smooth and the ground surface as horizontal.

In practice, a perfectly smooth wall is not realistic because some friction is developing between the wall and the ground.

The amount of shear stress, which can be mobilised at the wall-ground interface is determined by the wall-ground interface parameter $\delta$.

A concrete wall or steel sheet pile wall supporting sand or gravel may be assumed to have a design wall ground interface parameter $\delta = k \cdot \phi$.

Where:
- $k \leq 2/3$ for precast concrete or steel sheet piling
- $k = 1,0$ for concrete cast against soil

Values of the earth pressure coefficients may be taken from figures C.1.1 to C.1.4 for $K_a$ and C.2.1 to C.2.4 for $K_p$. These figures are taken from the Annex C of the Eurocode EN 1997-1 and are in a separate PDF file on the Moodle website.

7.6.1 Exercises

1. Looking at the figures of the Annex C of the Eurocode EN 1997-1, try to find how will change the earth pressure, if friction is taken into account in case of active pressure and passive pressure behind a wall retaining a horizontal ground surface. (answer this question on Moodle)

2. Find the total horizontal force acting, per linear meter of wall, on the precast concrete gravity wall (efficient drainage is present behind the wall)
7.7 Techniques used to resist lateral earth pressures

The most frequent structure used to retain soil is the retaining wall. Various types of retaining walls are described herebelow.

7.7.1 Gravity wall

Gravity walls depend on the weight of their mass (stone, concrete or other heavy material) to resist pressures from behind and will often have a slight inclination to improve stability by leaning back into the retained soil. For short landscaping walls, they are often made from mortarless stone or segmental concrete units.

7.7.2 Sheet pile wall

Sheet pile walls are often used in soft soils and tight spaces. Sheet pile walls are usually made out of steel sheet piles driven into the ground. As a rule of thumb: 1/3 third of the sheet pile is above ground, 2/3 below ground.

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7 Mur poids
8 battues
7.7.3 Cantilever wall

Cantilever walls are made from a relatively thin stem of steel-reinforced, cast-in-place concrete (often in the shape of an inverted T). These walls cantilever loads (like a beam) to a large, structural footing, converting horizontal pressures from behind the wall to vertical pressures on the ground below. Sometimes cantilevered walls include a counterfort on the back, to improve their stability against high loads.

These walls require rigid concrete footings below frost depth.

This type of wall uses much less material than a traditional gravity wall.

7.7.4 Secant or Tangent (also called contiguous) pile wall

Secant pile walls are formed by constructing intersecting reinforced concrete piles.

The piles are reinforced with either steel rebar or with steel beams and are constructed by either drilling under mud or augering.

Primary piles are installed first with secondary piles constructed in between primary piles once the latter gain sufficient strength.

Pile overlap is typically in the order of 8 cm.

In a tangent pile wall, there is no pile overlap as the piles are constructed flush to each other.

The main advantages of secant or tangent pile walls are:

1. Increased construction alignment flexibility.
2. Increased wall stiffness compared to sheet piles.

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9 Mur cantilever
10 Secant pile = pieu sécant
11 Tangent pile = pieu tangent
12 Barre d’armature
13 Forage à la tarière
14 à ras
3. Can be installed in difficult ground (cobbles/boulders).
4. Less noisy construction.

The main disadvantages of secant and tangent pile walls are:
1. Verticality tolerances may be hard to achieve for deep piles.
2. Total waterproofing is very difficult to obtain in joints.
3. Increased cost compared to sheet pile walls.

For a more comprehensive explanation of the building technique of such a wall, see the separate pdf file on the Moodle website.

7.7.5 Anchored wall\textsuperscript{15}

This version of wall uses cables or other stays anchored in the rock or soil behind it. Usually driven into the material with boring, anchors are then expanded at the end of the cable, either by mechanical means or often by injecting pressurized concrete, which expands to form a bulb in the soil. Technically complex, this method is very useful where high loads are expected, or where the wall itself has to be slender and would otherwise be too weak. The wall itself can be a sheet pile wall or secant pile wall.

\textbf{Note}: These two pictures were taken at the Guillemins station works in Liège.
7.7.6 Soil nailing

Soil nailing is a technique in which soil slopes, excavations or retaining walls are reinforced by the insertion of relatively slender elements - normally steel reinforcing bars. The bars are usually installed into a pre-drilled hole and then grouted into place or drilled and grouted simultaneously. They are usually installed untensioned at a slight downward inclination. A rigid or flexible facing (often sprayed concrete) or isolated soil nail heads may be used at the surface.

7.7.7 Soil-strengthened

A number of systems exist that do not simply consist of the wall itself, but reduce the earth pressure acting on the wall itself. These are usually used in combination with one of the other wall types, though some may only use it as facing (i.e. for visual purposes).

Gabion meshes

This type of soil strengthening, often also used without an outside wall, consists of wire mesh 'boxes' into which roughly cut stone or other material is filled. The mesh cages reduce some internal movement/forces, and also reduce erosive forces.

Mechanical stabilization

Mechanically stabilized earth is soil reinforced by layered horizontal mats (geosynthetics). Other options include steel straps, also layered. The wall face is often of precast concrete units that can tolerate some differential movement. The reinforced soil's mass, along with the facing, then acts as an improved gravity wall. The reinforced mass must be built large enough to retain the pressures from the soil behind it. Gravity walls usually must be a minimum of 50 to 60 percent as deep or thick as the height of the wall, and may have to be larger if there is a slope or surcharge on the wall.

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16 Sol cloué
17 injectée
18 gabions
19 Terre armée
7.7.8 Diaphragm wall

The continuous diaphragm wall (also referred to as slurry wall in the US) is a structure formed and cast in a slurry trench.

The trench excavation is initially supported by either bentonite or polymer based slurries that prevents soil incursions into the excavated trench. The term "diaphragm walls" refers to the final condition when the slurry is replaced by tremied concrete that acts as a structural system either for temporary excavation support or as part of the permanent structure. This construction sequence is illustrated in Figure 19.

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20 Paroi moulée
21 La bentonite est un argile
7.7.9 Timbered trench

This kind of wall retains the soil by means of struts and sheetings. It is widely used for trenches.

Figure 20: timbered trench

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22 Tranchée blindée
23 entretoise
24 coffrage