Chapitre 6. SOIL STRENGTH

Soils are essentially frictional materials. They are comprised of individual particles that can slide and roll relative to one another. In the discipline of soil mechanics it is generally assumed that the particles are not cemented.

One consequence of the frictional nature is that the strength depends on the effective stresses in the soil. As the effective stresses increase with depth, so in general will the strength.

The strength will also depend on whether the soil deformation occurs under fully drained conditions, constant volume (undrained) conditions, or with some intermediate state of drainage. In each case different excess pore pressures will occur resulting in different effective stresses, and hence different strengths. In assessing the stability of soil constructions analyses are usually performed to check the short term (undrained) and long term (fully drained) conditions.

6.1 Principal planes and principal stresses

The study of the stress distribution around a point in a continuous demonstrates that there are three orthogonal planes, called principal planes, where the stresses are normal and called the main stresses. They are designated by:

- $\sigma_1$ for the major principal stress,
- $\sigma_2$ for the minor principal stress,
- $\sigma_2$ for the intermediary principal stress.

The applications, in soil mechanics, can be reduced to:

- plane problems (the intermediary principal stress $\sigma_2$ is perpendicular to the plane containing $\sigma_1$ and $\sigma_3$)
- axysymmetric (3D revolution) problems ($\sigma_2 = \sigma_3$ and a meridian contains $\sigma_1$ and $\sigma_3$).

It follows that the study of the distribution stresses in the $\sigma_1$-$\sigma_3$ plays an important role in soil mechanics and will be discussed below.

6.2 Mohr Circle

The Mohr circle construction enables the stresses acting in different directions at a point on a plane to be determined, provided that the normal and shear stresses are known on any two orthogonal planes.
planes. The Mohr circle construction is very useful in Soil Mechanics as many practical situations can be approximated as plane strain problems.

The sign convention is different to that used in Structural Analysis because for soils it is conventional to take the compressive stresses as positive.

Sign convention:
- Compressive normal stresses are positive
- Anti-clockwise shear stresses are positive (from inside soil element)
- Angles measured anti-clockwise positive

Let us consider the stresses acting on an element of soil delimited by three planes, two of them being orthogonal:

The equilibrium of the solid gives:

\[ \tau_n = \frac{\sigma_y - \sigma_x}{2} \sin(2\theta) - \tau_{xy} \cos 2\theta \]
\[ \sigma_n = \frac{\sigma_y + \sigma_x}{2} + \frac{\sigma_y - \sigma_x}{2} \cos(2\theta) + \tau_{xy} \sin 2\theta \]

**Figure 1** : Mohr circle relations

**Figure 2** : Mohr circle graphical representation of stresses at one point

6.2.1 **Pole Method of Finding Stresses on a Plane**

1. Each stress state at one point on a plane is represented by a point M on the Mohr circle.
2. From that point, draw a parallel to the plane on which act the stresses (\(\sigma, \tau\)). The intersection with the Mohr circle is called the pole point\(^1\) P,

\(^1\) Anti-horloégique (dans le sens contraire des aiguilles d'une montre)
3. To know the stresses acting on a plane EF, draw a parallel to the plane EF from the pole point. The intersection with the circle gives point Q. This point Q represents the stress state on EF.

4. The major and minor principal stresses $\sigma_1$ and $\sigma_3$ are found at the intersection with the $\sigma$ axis (by definition, a principal stress is a stress having only a normal component).

### 6.3 Mohr-Coulomb failure$^3$ criterion

Mohr and Coulomb found that failure in a soil will occur when the stresses $(\sigma'_\alpha, \tau'_\alpha)$ on any plane are such that:

$$|\tau'_\alpha| \geq c' + \sigma'_\alpha \tan \phi'$$ (shaded area on Fig.3)

where

- $\tau'_\alpha = \text{effective shear stress at failure}$
- $c' = \text{effective (drained) cohesion}$
- $\phi' = \text{effective friction angle}^4$
- $\sigma'_\alpha = \text{effective normal stress at failure}$

![Figure 3: Mohr-Coulomb failure criterion](image)

On Fig.3, if we take the plane on which acts $\sigma'_1$ as the horizontal reference (we always can turn the figure to see it horizontal), the pole point P is at the intersection of an horizontal line and the circle, thus the point $(\sigma'_3,0)$. From P a line inclined with an angle $\alpha$ relatively to the reference plane (on which acts $\sigma'_1$), gives the point $(\sigma'_\alpha, \tau'_\alpha)$. If we can find any angle $\alpha$ such that $(\sigma'_\alpha, \tau'_\alpha)$ is located in the shaded area, there is failure. The limit state of equilibrium in one point is thus when the Mohr circle for that point is tangent to the Mohr-Coulomb lines.

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$^2$ centre de rayonnement  
$^3$ rupture  
$^4$ angle de frottement interne
Granular soils like gravel, sand, or silt (coarse grained soil) with little or no clay content, exhibit no effective cohesion ($c' \approx 0$) and high effective friction angle. Granular soils crumble easily when dry.

Cohesive soil like clay (fine grained soil), or with a high clay content, exhibit significant effective cohesion and low effective friction angle. Cohesive soils do not crumble, can be excavated with vertical excavation slopes, and are plastic when moist.

The Mohr-Coulomb criterion is an empirical criterion, and the failure locus is only locally linear. Extrapolation outside the range of normal stresses for which it has been determined is likely to be unreliable.

### 6.3.1 Exercise

1. Starting from Fig.3, try to find, in terms of $\phi'$, the values of:
   - the angle of the failure plane relatively to the plane on which acts $\sigma'_1$
   - the angle of the failure plane relatively to the plane on which acts $\sigma'_3$

### 6.4 Strength Tests

The engineering strength of soil materials is often determined from tests in either the shear box apparatus or the triaxial apparatus.

#### 6.4.1 The Shear Box Test

The soil is sheared along a predetermined plane by placing it in a box and then moving the top half of the box relative to the bottom half. The box may be square or circular in plan and of any size, however, the most common shear boxes are square, 60 mm x 60 mm.

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5. Sols pulvérulents
6. S’effriter
7. Sols cohérents
8. essai de cisaillement a la boite (shear box = boite de cisaillement (aussi appelée boite de Casagrande)}
A load normal to the plane of shearing may be applied to a soil specimen through the lid\textsuperscript{9} of the box. Provision is made for porous plates to be placed above and below the soil specimen. These enable drainage to occur which is necessary if a specimen is to be consolidated under a normal load, and if a specimen is to be tested in a fully drained state. The soil specimen may be submerged, by filling the containing vessel\textsuperscript{10} with water, to prevent the specimens from drying out. Undrained tests may be carried out, but in this case solid spacer blocks rather than the porous disks must be used.

**Notation**

\begin{align*}
N & = \text{Normal Force} \\
F & = \text{Tangential (Shear) Force} \\
\sigma_n & = \text{Normal Stress} \\
\tau & = \frac{F}{A} = \text{Shear Stress} \\
A & = \text{Cross-sectional area of shear plane} \\
dx & = \text{Horizontal displacement} \\
dy & = \text{Vertical displacement}
\end{align*}

Usually only relatively slow drained tests are performed in shear box apparatus. For clays the rate of shearing must be chosen to prevent excess pore pressures building up. For freely draining sands and gravels tests can be performed quickly. Tests on sands and gravels are usually performed dry as it is found that water does not significantly affect the (drained) strength.

Provided there are no excess pore pressures the pore pressure in the soil will be approximately zero and the total and effective stresses will be identical. That is, \( \sigma_n = \sigma'_n \)

The failure stresses thus define an effective stress failure envelope from which the effective (drained) strength parameters \( c', \phi' \) can be determined.

**Typical test results**
The shear box is far from ideal. Disadvantages of the test include:

- Non-uniform deformations and stresses.
- There are no facilities for measuring pore pressures in the shear box and so it is not possible to determine effective stresses from undrained tests.

However, it has the advantage of its simplicity.

\[ \tau' = c' + \sigma'_n \tan \phi' \] (Fig. 6)

From this, the effective (drained) strength parameters \( c' \) and \( \phi' \) can be determined.

The shear box is far from ideal. Disadvantages of the test include:

- Normal force

\[ \sigma_1' = \sigma_2' = \sigma_3' = \frac{F}{A} \] (Fig. 6)

In practice the shear box is used to get quick and crude estimates of the failure parameters.
6.4.2 Exercise

2 A saturated compacted gravel was tested in a large shear box, 300 mm x 300 mm in plan. What properties of the gravel can be deduced from the following results?

<table>
<thead>
<tr>
<th>Normal load (N)</th>
<th>Peak Shear Load (N)</th>
</tr>
</thead>
<tbody>
<tr>
<td>4500</td>
<td>4500</td>
</tr>
<tr>
<td>9200</td>
<td>7890</td>
</tr>
<tr>
<td>13800</td>
<td>11200</td>
</tr>
</tbody>
</table>

6.4.3 The Triaxial Test

The triaxial test is carried out in a cell and is so named because three principal stresses are applied to the soil sample during the test. A diagram of a typical triaxial cell is shown on Fig.7.

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11 cellule
A cylindrical soil specimen as shown is placed inside a latex rubber\textsuperscript{12} sheath\textsuperscript{13} which is sealed to a top and base cap by rubber O-rings. For drained tests, or undrained tests with pore pressure measurement, porous disks are placed at the bottom, and sometimes at the top of the specimen. For tests where consolidation of the specimen is to be carried out, filter paper drains may be provided around the outside of the specimen in order to speed up the consolidation process.

Pore pressure generated inside the specimen during testing may be measured by means of pressure transducers\textsuperscript{14}.

**Stresses**

The water pressure inside the confining cell induces three equal principal stresses in the soil sample ($\sigma$, see Fig.8). The axial principal stress is increased by applying an additional pressure through a loading ram\textsuperscript{15} through the top of the cell.

![Figure 8: Stresses in the Triaxial Test](image)

From vertical equilibrium we have

$$\sigma_a = \sigma_r + \frac{F}{A}$$

The term $F/A$ is known as the deviator stress\textsuperscript{16}. The deviator load $F$ is increased until failure.

As the axial and radial stresses are principal stresses, and $\sigma_a > \sigma_r$, we can write:

- $\sigma_1 = \sigma_a$
- $\sigma_3 = \sigma_r$

\textsuperscript{12} caoutchouc \hfill \textsuperscript{13} gaine \hfill \textsuperscript{14} capteurs \hfill \textsuperscript{15} piston \hfill \textsuperscript{16} Déviateur des contraintes
**Test procedures**

There are many test variations. The most used are reproduced below. The choice of the kind of triaxial test should be made taking into account that the conditions reproduced during a laboratory triaxial test should be the same as those anticipated in the field for the particular investigation under consideration.

unconsolidated undrained test\(^{17}\).

Cell pressure applied without allowing drainage. Then keeping cell pressure constant increase deviator stress to failure without drainage.

consolidated undrained test\(^{18}\).

Drainage allowed during cell pressure application. Then, without allowing further drainage, increase the deviator stress keeping cell pressure constant as for unconsolidated undrained test.

consolidated drained test\(^{19}\).

Similar to consolidated undrained except that, as deviator stress is increased, drainage is permitted. The rate of loading must be slow enough to ensure no excess pore pressures develop.

**Example**

Enumerate the types of laboratory triaxial test you would specify to be carried out in connection with the following field problems:

(a) the stability of a clay foundation of an embankment, the rate of construction being such that some consolidation of the clay occurs;

(b) the initial stability of a footing on saturated clay;

(c) the long-term stability of a slope in stiff fissured clay.

Answer:

(a) Since there is some consolidation during construction of the embankment, a consolidated undrained triaxial test with pore water pressure measurements would be appropriate in this case.

(b) A footing on saturated clay will initially increase the pore water pressure of the clay and only gradually, as consolidation occurs, will the effective stresses increase. The appropriate test in this case, therefore, would be an unconsolidated undrained triaxial test.

(c) The long-term stability of a slope in a stiff fissured clay would depend on the effects of consolidation and water seepage. A consolidated drained test would give the necessary information about the long-term shearing resistance of the clay.

\(^{17}\) Essai non consolidé non drainé

\(^{18}\) Essai consolidé non drainé

\(^{19}\) Essai consolidé drainé
The triaxial test has many advantages over the shear box test:
- Specimens are subjected to uniform stresses and strains
- Drained and undrained tests can be performed
- Pore water pressures can be measured in undrained tests
- Different combinations of confining and axial stress can be applied

Whereas the shear box test directly gives the Mohr-Coulomb lines, the triaxial test needs us to draw the Mohr circles corresponding to each test failure. The Mohr-Coulomb lines are tangent to these Mohr circles.

![Figure 8: Stresses in the Triaxial Test](image)

### 6.4.4 Exercises

3. Undrained triaxial tests with pore pressure measurement have been performed on three samples of a particular soil, after consolidation to different cell pressures. What information (strength parameters) can be obtained from the results given below?

<table>
<thead>
<tr>
<th>Cell pressure (kPa)</th>
<th>Failure Deviator Stress (kPa)</th>
<th>Failure Pore Pressure (kPa)</th>
</tr>
</thead>
<tbody>
<tr>
<td>24</td>
<td>31</td>
<td>12</td>
</tr>
<tr>
<td>48</td>
<td>76</td>
<td>18</td>
</tr>
<tr>
<td>72</td>
<td>104</td>
<td>30</td>
</tr>
</tbody>
</table>

4. A soil has an apparent cohesion $c' = 5$ kPa and an angle of friction $\phi' = 35^\circ$. A sample of this soil is consolidated in a triaxial cell by applying a cell pressure $\sigma_3 = 70$ kPa. The sample is then failed by increasing the axial stress under undrained conditions ($\sigma_3$ remains constant).

   Calculate the axial stress at failure if the pore pressure at failure $u = 20$ kPa.