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Chapitre 10. Flow Nets[1]

10.1 Introduction

Let us consider a state of plane seepage as for example in the earth dam shown in Figure 1.

[1] Réseaux d'écoulement

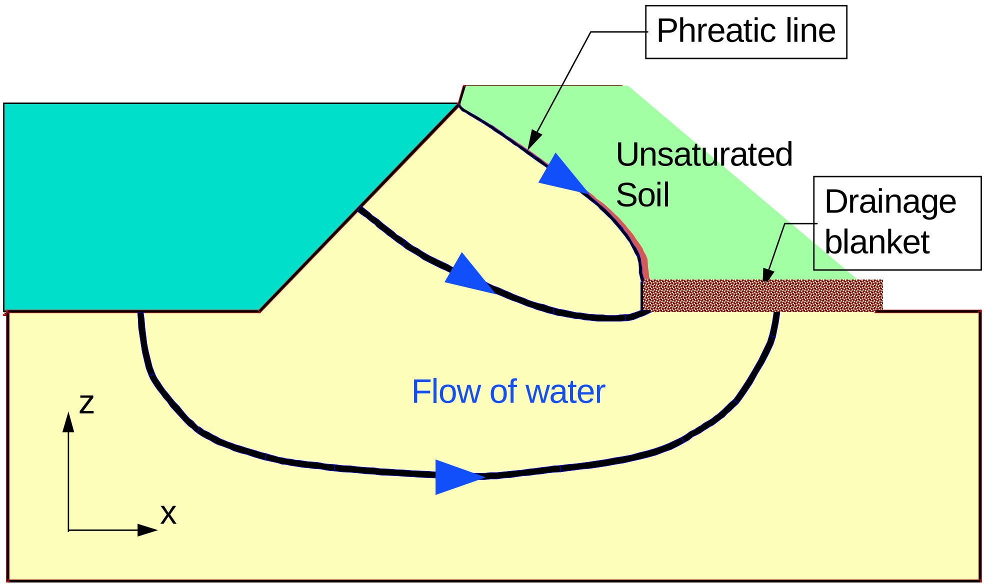


Fig. 1 Flow through an earth dam

For an isotropic material, the head *h* of the water flowing into the soil satisfies Laplace's equations, thus analysis involves the solution of:

$$\frac{\partial^2 h}{\partial x^2}+\frac{\partial^2 h}{\partial z^2}=0$$

subject to certain boundary conditions.

10.2 Representation of Solution

At every, point (x,z) where there is flow there will be a value of head h(x,z). In order to represent these values we draw contours of equal head (red lines) as shown on Figure 2.

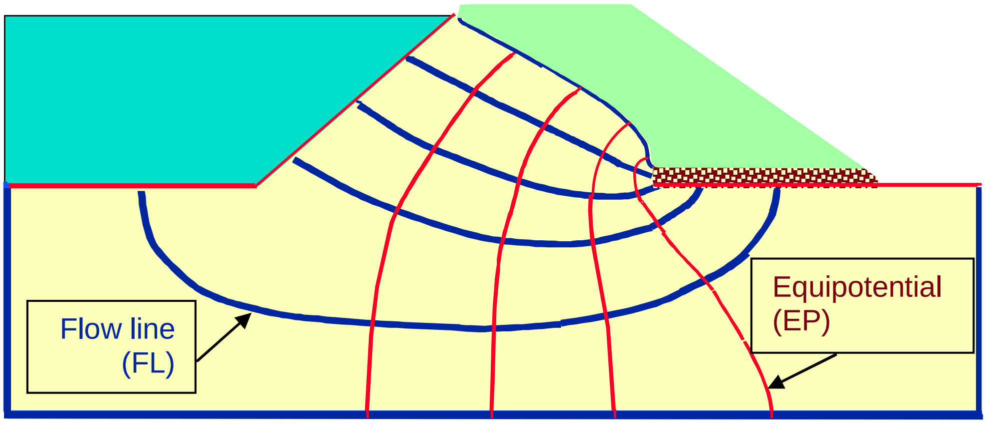


Fig.2 Flow lines and equipotentials

These lines are called equipotentials[2]. On an equipotential (EP). by definition:

$$h(x,z) = \text{constant}$$

[2] équipotentielles

It is also useful in visualising the flow in a soil to plot the flow lines[3] (blue lines noted FL), these are lines that are tangential to the flow at a given point and are illustrated in Figure 2.

It can be seen from Fig. (2) that the flow lines and equipotentials are orthogonal.

10.3 Some Geometric Properties of Flow Nets

Let us consider another example of seepage under a sheet pile wall. The flow net is represented on figure 3.

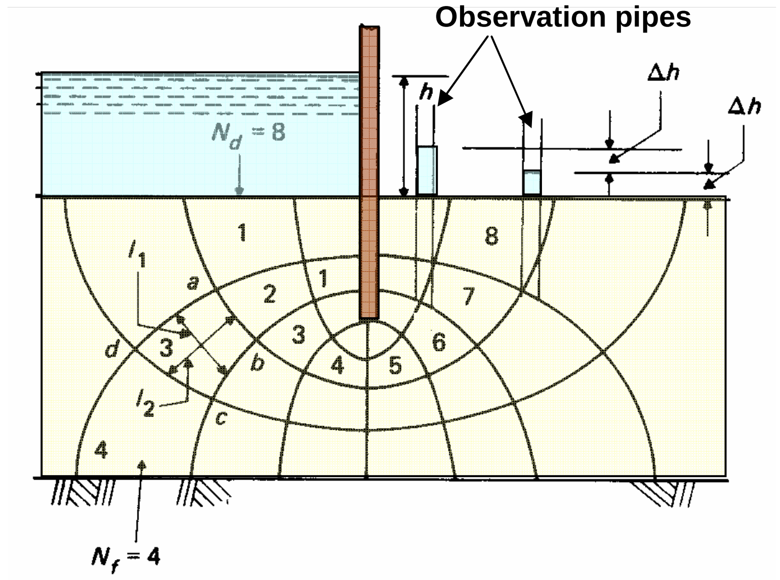


Figure 3

On Fig. 3, each interval between two equipotentials corresponds to a head loss Δh equal to $1/N_d$ of the total head loss h through the soil :

$$\Delta h = h / N_d$$

Where N_d = Total number of equipotentials.

Consider a pair of flow lines, clearly the flow through this flow tube must be constant and so as the tube narrows the velocity must increase.

Let us consider the flow through *abcd* delimited by two flow lines and two equipotentials.

The hydraulic gradient is :

$$i = \Delta h / l_1 = h / (N_d . l_1)$$

Where l_1 is the distance between the two equipotentials.

---

[3] Lignes de courant

Let l_2 be the distance between the two flow lines.

Applying Darcy's law, the velocity in the tube *abcd* is:

$$v = k \cdot i = k \cdot h / (N_d \cdot l_1)$$

The flow passing into *abcd*, per m width of soil, is :

$$\Delta q_{abcd} = \text{Area x velocity} = l_2 \cdot 1 \cdot k \cdot h / (N_d \cdot l_1)$$

If we draw the flow net taking $l_2 = l_1$ (a "squared" mesh net is more convenient to draw : it is possible to draw an inscribed circle, see Fig.4), $\Delta q_{abcd} = k \cdot h / N_d$ (the flow through any quadrilateral of the flow net is thus the same as the one through *abcd*).

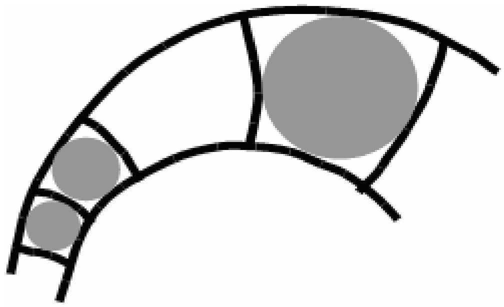


Figure 4 Inscribing Circles in a Flow Net

The total flow will then be equal to :

$$Q = N_f \cdot \Delta q = k \cdot h \cdot N_f / N_d$$

Where N_f = Number of flow tubes.

From that equation, one can see that the flow is function of the ratio N_f / N_d and thus if the flow net is refined by dividing each cell in four smaller cells, the ratio will remain unchanged. That means that Q is independent of the refinement of the flow net! It is thus easy to determine quickly an estimate of the flow of water passing under a dam or wall.

To calculate quantities of interest, that is the flow and pore water pressures, a flow net must be drawn.

The flow net must consist of two families of orthogonal lines that ideally define a square mesh, and that also satisfy the boundary conditions.

The three most common boundary conditions are discussed below.

10.4 Common boundary conditions

10.4.1 Submerged Soil Boundary = Equipotential

Consider the submerged soil boundary shown in Figure 5

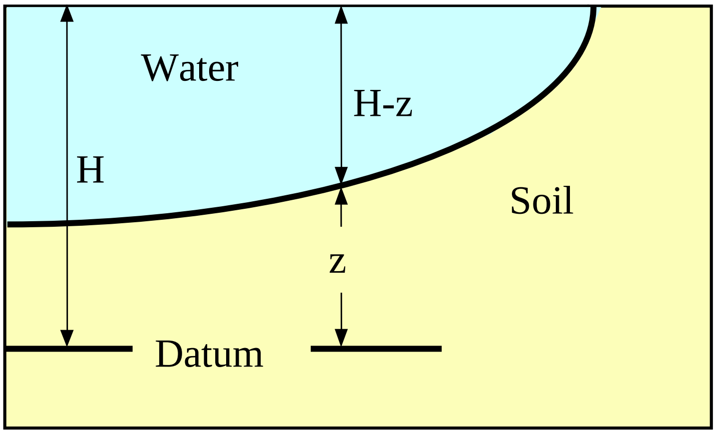


Figure 5 Equipotential boundary

The head at the indicated position is calculated as follows:

$$h = \frac{u_w}{\gamma_w} + z$$

now

$$u_w = \gamma_w (H - z)$$

so

$$h = \frac{(H - z)\gamma_w}{\gamma_w} + z = H$$

That is, the head is constant for any value of z, which is by definition an equipotential. Alternatively, this could have been determined by considering imaginary observation pipes placed at the soil boundary, as for every point the water level in the standpipe would be the same as the water level.

A consequence of this is that all the flow lines arrive perpendicularly to a submerged soil boundary.

The upstream face of the dam shown in Figures 1 and 2 is an example of this situation.

10.4.2 Impermeable Boundary = Flow Line

At a boundary between permeable and impermeable material the velocity normal to the boundary must be zero since otherwise there would be water flowing into or out of the impermeable material, this is illustrated in Figure 6.

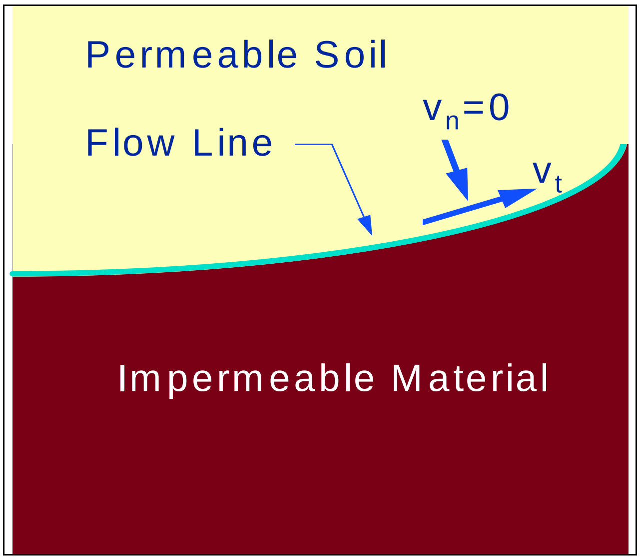


Figure 6 Flow line boundary

A consequence of this is that all the equipotentials arrive perpendicularly to an impermeable soil boundary.
The phreatic surface shown in Figures 1 and 2 is also a flow line marking the boundary of the flow net. A phreatic surface is also a line of constant (zero) pore water pressure as discussed below.

10.4.3 Line of Constant Pore Water Pressure

Sometimes a portion of saturated soil is in contact with air and so the pore water pressure of the water just beneath that surface is atmospheric. The phreatic surface shown in Figure 7 below is an example of such a condition. We can show from the expression for head in terms of pore water pressure that equipotentials intersecting a line of constant pore water pressure do so at equal vertical intervals as follows:

$$h = \frac{u_w}{\gamma_w} + z$$

thus

$$\Delta h = \frac{\Delta u_w}{\gamma_w} + \Delta z$$

now $\Delta u_w = 0$

and so

$$\Delta h = \Delta z$$

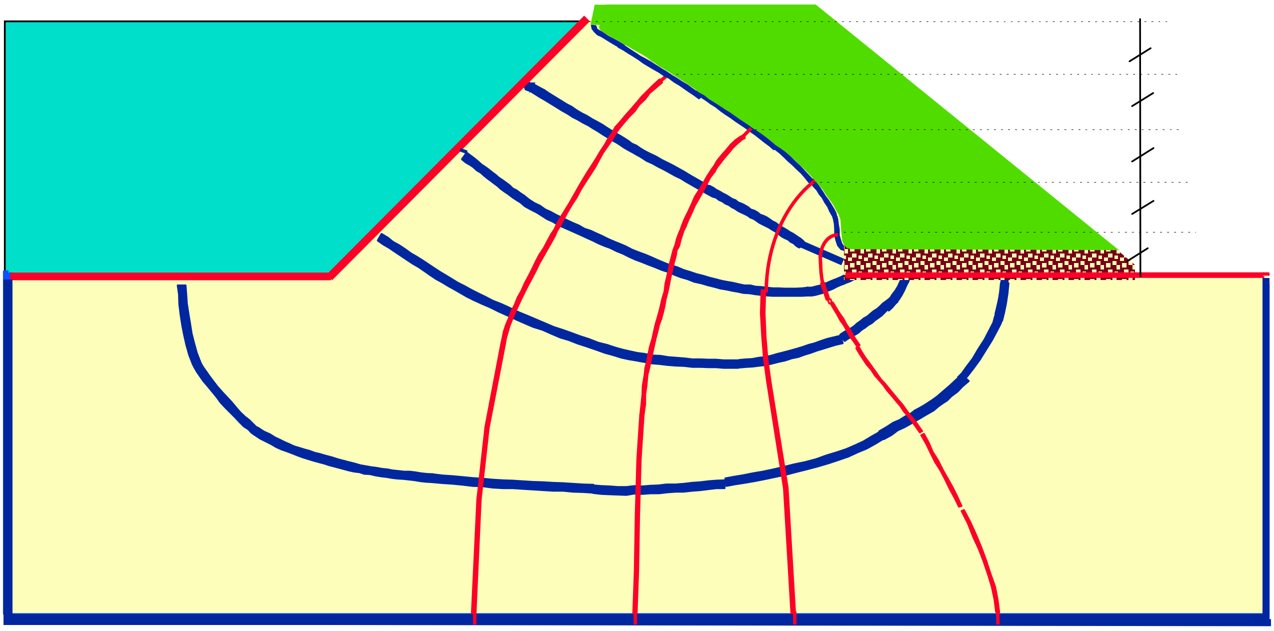


Figure 7 Constant pore water pressure boundary

10.5 Procedure for Drawing Flow Nets

1. Mark all boundary conditions. Determine the head at the inlet and outlet of the flow net.

2. Draw a coarse net which is consistent with the boundary conditions and which has orthogonal equipotential and flow lines. (It is usually easier to start by drawing the flow lines).

3. Modify the net so that it meets the conditions outlined above and so that the mesh located between adjacent flow lines and equipotentials are square (you could draw an inscribed circle).

Refine the flow net by repeating step 3.

10.6 Calculation of Quantities of Interest from Flow Nets

10.6.1 Calculation of flow

We have seen at 10.3 that, when the flow net has been drawn so that the elemental rectangles are approximately square, the total flow is equal to :

$$Q = N_f \cdot \Delta q = k \cdot h \cdot N_f / N_d$$

It should be noted in the development of this formula it was assumed that each flow tube was of unit width and so the flow is given per unit width (into the page).
Let us come back to the earth dam example.

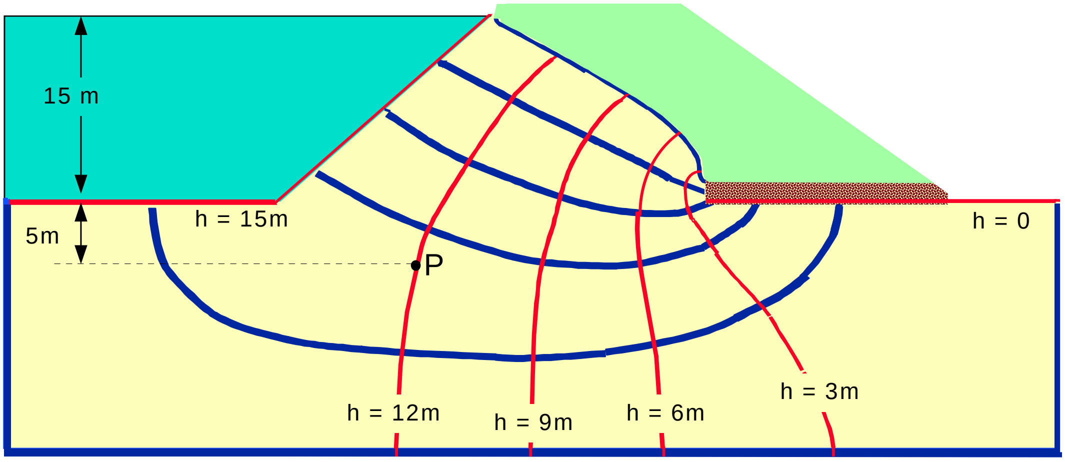


Figure 8 Value of Head on Equipotentials

In this example, we have N_f = the number of flow tubes = 5, and N_d = the number of equipotential drops = 5.
Suppose that the permeability of the underlying soil is k=10^{-5} m/sec (typical of a fine sand or silt) then the flow per unit width of dam is:

$$Q = 15 \times 10^{-5} \quad \text{m}^3\text{/sec (per m width)}$$

and if the dam is 25m wide the total flow under the dam:

$$Q = 25 \times 15 \times 10^{-5} \quad \text{m}^3\text{/sec}$$

10.6.2 Calculation of Pore Water Pressure

The pore water pressure at any point can be found using the expression

$$h = \frac{u_w}{\gamma_w} + z$$

Now referring to Fig. 8 suppose that we wish to calculate the pore water pressure at the point P. Taking the datum to be at the base of the dam it can be seen that z = - 5m and so:

$$u_w = [12 - (-5)]\gamma_w = 17\gamma_w$$

10.6.3 Exercise

1. A sheet pile wall is driven to a depth of 6m into a permeable sand layer (k = 6.10^{-3} mm/s) of 13.5 m thickness lying on an impermeable layer. The water on one side of the wall is at a height of 4.5m, while on the other side, pumps maintain the water level at ground level. To design the pumping system, draw, by hand, the flow net and estimate the flow under the wall in m³/day.

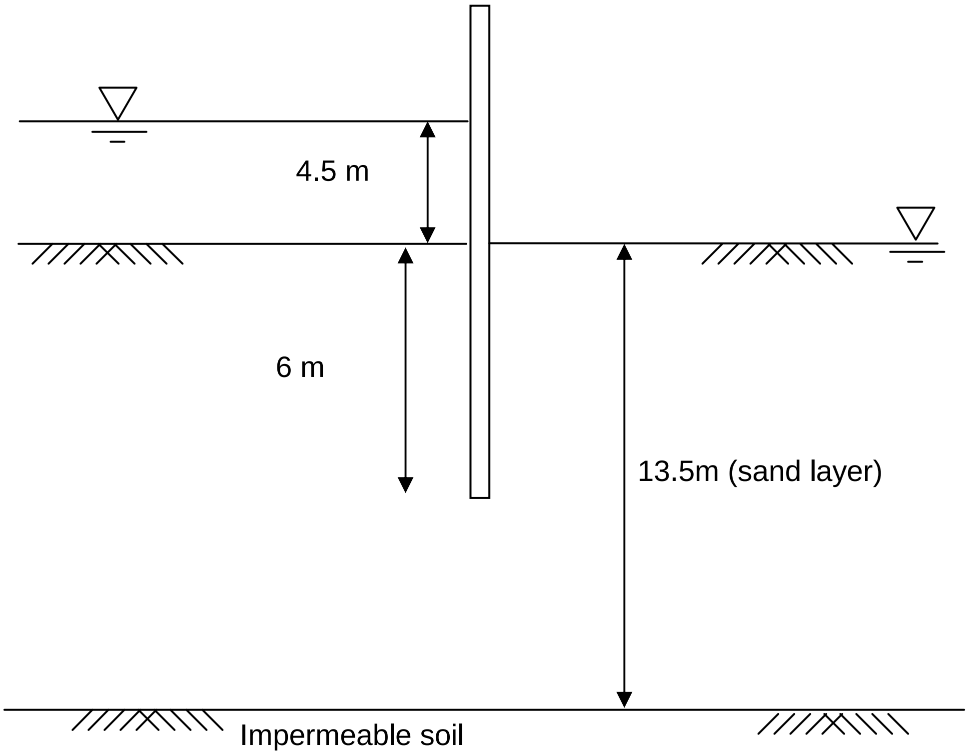
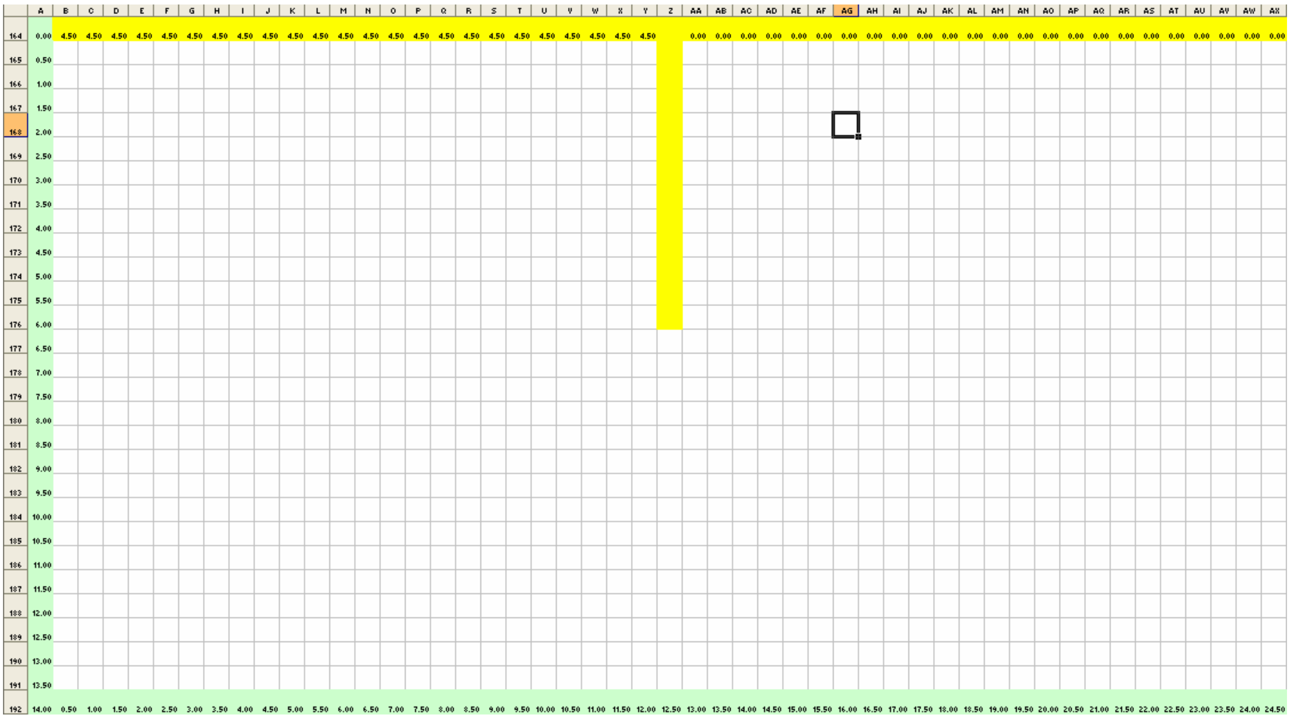


Figure 9

To check your flow net drawn by hand, you can use FDSOLVER.XLA, a excel macro using the Finite Difference Method. That macro is available on the course Moodle website, together with an install note[4] and an help file containing examples. That macro can be also be useful in other domains like heat diffusion, electric potential, fluid flow, vibrations,…

The steps to follow are explained in the help file. You should "draw" the model like this :

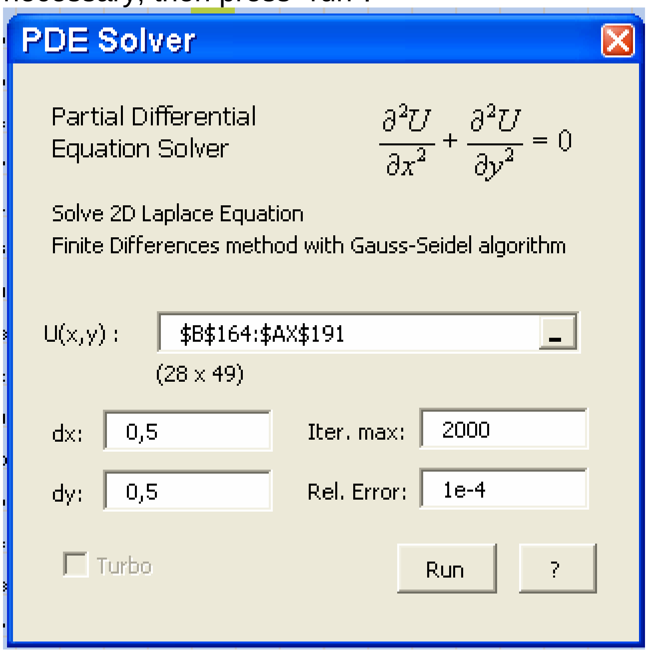
[4] An update to the install note has been added by a 2011-1012 student, François Diffels, for Excel 2007 and 2010. Thanks François!



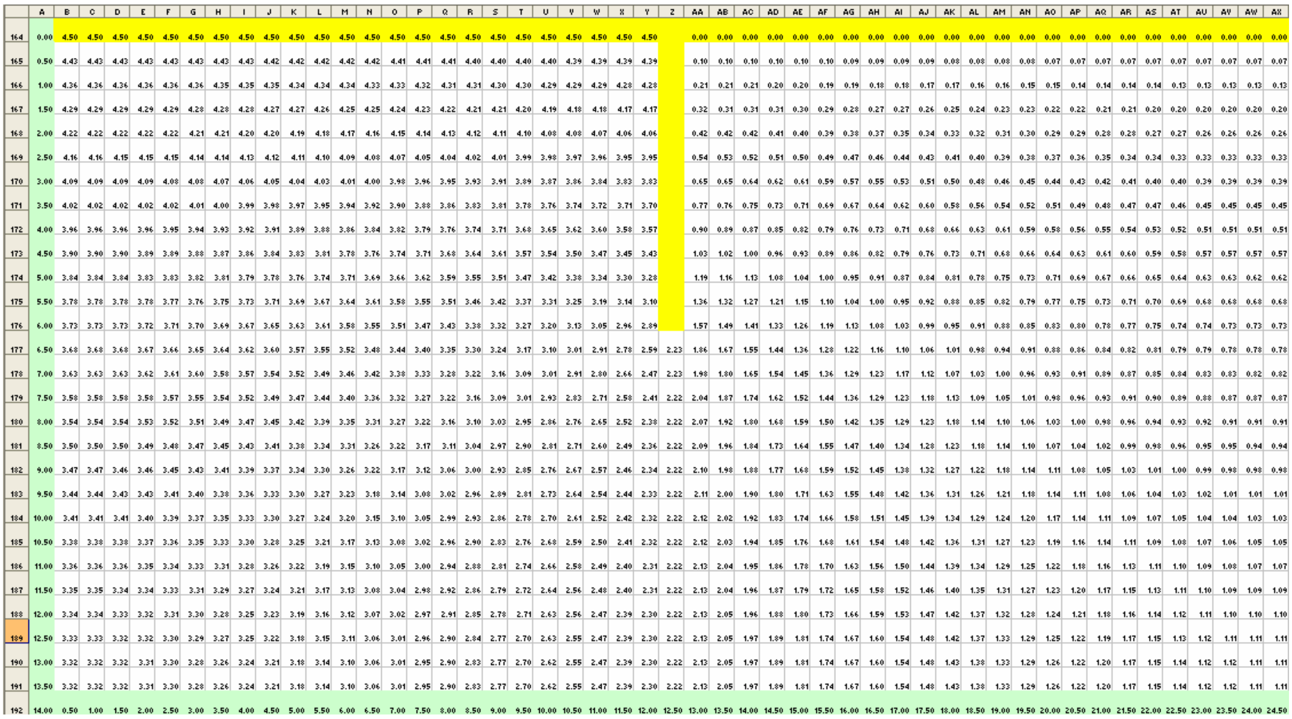
The light green cells (left and bottom) are the y and x scales (0.5m step) (not mandatory, but it helps to draw the model to scale.

The yellow cells are either head specified boundaries (the top lines at either side of the wall), or an impermeable boundary (the wall itself).

To solve the system and get the head values at any point (cell), just select the area with the white and yellow cells, then select Tools, FDSOLVER, 2D-LAPLACE, then specify the scale if necessary, then press "run".



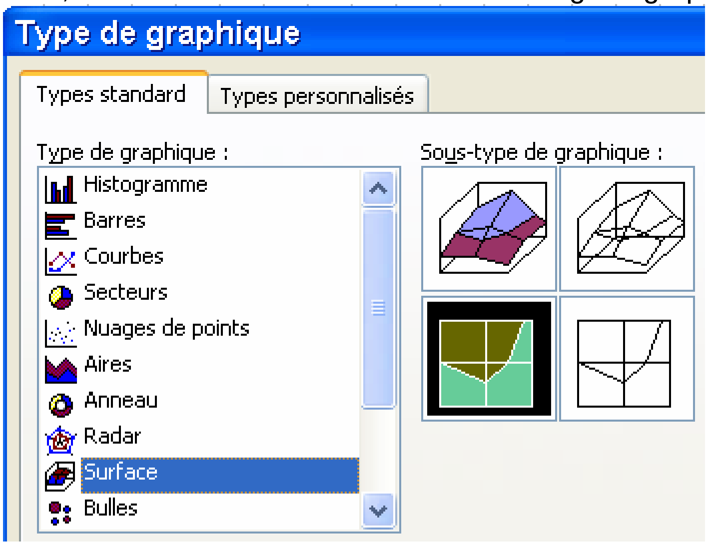
The numerical results appear after a while :



	A	B	C	D	E	F	G	H	I	J	K	L	M	N	O	P	Q	R	S	T	U	V	W	X	Y	Z
164	0.00	4.50	4.50	4.50	4.50	4.50	4.50	4.50	4.50	4.50	4.50	4.50	4.50	4.50	4.50	4.50	4.50	4.50	4.50	4.50	4.50	4.50	4.50	4.50	4.50	
165	0.50	4.43	4.43	4.43	4.43	4.43	4.43	4.43	4.43	4.42	4.42	4.42	4.42	4.42	4.41	4.41	4.41	4.40	4.40	4.40	4.40	4.39	4.39	4.39	4.39	
166	1.00	4.36	4.36	4.36	4.36	4.36	4.36	4.35	4.35	4.35	4.34	4.34	4.34	4.33	4.33	4.32	4.31	4.31	4.30	4.30	4.29	4.29	4.29	4.28	4.28	
167	1.50	4.29	4.29	4.29	4.29	4.29	4.28	4.28	4.28	4.27	4.27	4.26	4.25	4.25	4.24	4.23	4.22	4.21	4.21	4.20	4.19	4.18	4.18	4.17	4.17	
168	2.00	4.22	4.22	4.22	4.22	4.22	4.21	4.21	4.20	4.20	4.19	4.18	4.17	4.16	4.15	4.14	4.13	4.12	4.11	4.10	4.08	4.08	4.07	4.06	4.06	
169	2.50	4.16	4.16	4.15	4.15	4.15	4.14	4.14	4.13	4.12	4.11	4.10	4.09	4.08	4.07	4.05	4.04	4.02	4.01	3.99	3.98	3.97	3.96	3.95	3.95	
170	3.00	4.09	4.09	4.09	4.09	4.08	4.08	4.07	4.06	4.05	4.04	4.03	4.01	4.00	3.98	3.96	3.95	3.93	3.91	3.89	3.87	3.86	3.84	3.83	3.83	
171	3.50	4.02	4.02	4.02	4.02	4.02	4.01	4.00	3.99	3.98	3.97	3.95	3.94	3.92	3.90	3.88	3.86	3.83	3.81	3.78	3.76	3.74	3.72	3.71	3.70	
172	4.00	3.96	3.96	3.96	3.96	3.95	3.94	3.93	3.92	3.91	3.89	3.88	3.86	3.84	3.82	3.79	3.76	3.74	3.71	3.68	3.65	3.62	3.60	3.58	3.57	
173	4.50	3.90	3.90	3.90	3.89	3.89	3.88	3.87	3.86	3.84	3.83	3.81	3.78	3.76	3.74	3.71	3.68	3.64	3.61	3.57	3.54	3.50	3.47	3.45	3.43	
174	5.00	3.84	3.84	3.84	3.83	3.83	3.82	3.81	3.79	3.78	3.76	3.74	3.71	3.69	3.66	3.62	3.59	3.55	3.51	3.47	3.42	3.38	3.34	3.30	3.28	
175	5.50	3.78	3.78	3.78	3.78	3.77	3.76	3.75	3.73	3.71	3.69	3.67	3.64	3.61	3.58	3.55	3.51	3.46	3.42	3.37	3.31	3.25	3.19	3.14	3.10	
176	6.00	3.73	3.73	3.73	3.72	3.71	3.70	3.69	3.67	3.65	3.63	3.61	3.58	3.55	3.51	3.47	3.43	3.38	3.32	3.27	3.20	3.13	3.05	2.96	2.89	
177	6.50	3.68	3.68	3.68	3.67	3.66	3.65	3.64	3.62	3.60	3.57	3.55	3.52	3.48	3.44	3.40	3.35	3.30	3.24	3.17	3.10	3.01	2.91	2.78	2.59	2.23
178	7.00	3.63	3.63	3.63	3.62	3.61	3.60	3.58	3.57	3.54	3.52	3.49	3.46	3.42	3.38	3.33	3.28	3.22	3.16	3.09	3.01	2.91	2.80	2.66	2.47	2.23
179	7.50	3.58	3.58	3.58	3.58	3.57	3.55	3.54	3.52	3.49	3.47	3.44	3.40	3.36	3.32	3.27	3.22	3.16	3.09	3.01	2.93	2.83	2.71	2.58	2.41	2.22
180	8.00	3.54	3.54	3.54	3.53	3.52	3.51	3.49	3.47	3.45	3.42	3.39	3.35	3.31	3.27	3.22	3.16	3.10	3.03	2.95	2.86	2.76	2.65	2.52	2.38	2.22
181	8.50	3.50	3.50	3.50	3.49	3.48	3.47	3.45	3.43	3.41	3.38	3.34	3.31	3.26	3.22	3.17	3.11	3.04	2.97	2.90	2.81	2.71	2.60	2.49	2.36	2.22
182	9.00	3.47	3.47	3.46	3.46	3.45	3.43	3.41	3.39	3.37	3.34	3.30	3.26	3.22	3.17	3.12	3.06	3.00	2.93	2.85	2.76	2.67	2.57	2.46	2.34	2.22
183	9.50	3.44	3.44	3.43	3.43	3.41	3.40	3.38	3.36	3.33	3.30	3.27	3.23	3.18	3.14	3.08	3.02	2.96	2.89	2.81	2.73	2.64	2.54	2.44	2.33	2.22
184	10.00	3.41	3.41	3.41	3.40	3.39	3.37	3.35	3.33	3.30	3.27	3.24	3.20	3.15	3.10	3.05	2.99	2.93	2.86	2.78	2.70	2.61	2.52	2.42	2.32	2.22
185	10.50	3.38	3.38	3.38	3.37	3.36	3.35	3.33	3.30	3.28	3.25	3.21	3.17	3.13	3.08	3.02	2.96	2.90	2.83	2.76	2.68	2.59	2.50	2.41	2.32	2.22
186	11.00	3.36	3.36	3.36	3.35	3.34	3.33	3.31	3.28	3.26	3.22	3.19	3.15	3.10	3.05	3.00	2.94	2.88	2.81	2.74	2.66	2.58	2.49	2.40	2.31	2.22
187	11.50	3.35	3.35	3.34	3.34	3.33	3.31	3.29	3.27	3.24	3.21	3.17	3.13	3.08	3.04	2.98	2.92	2.86	2.79	2.72	2.64	2.56	2.48	2.40	2.31	2.22
188	12.00	3.34	3.34	3.33	3.32	3.31	3.30	3.28	3.25	3.23	3.19	3.16	3.12	3.07	3.02	2.97	2.91	2.85	2.78	2.71	2.63	2.56	2.47	2.39	2.30	2.22
189	12.50	3.33	3.33	3.32	3.32	3.30	3.29	3.27	3.25	3.22	3.18	3.15	3.11	3.06	3.01	2.96	2.90	2.84	2.77	2.70	2.63	2.55	2.47	2.39	2.30	2.22
190	13.00	3.32	3.32	3.32	3.31	3.30	3.28	3.26	3.24	3.21	3.18	3.14	3.10	3.06	3.01	2.95	2.90	2.83	2.77	2.70	2.62	2.55	2.47	2.39	2.30	2.22
191	13.50	3.32	3.32	3.32	3.31	3.30	3.28	3.26	3.24	3.21	3.18	3.14	3.10	3.06	3.01	2.95	2.90	2.83	2.77	2.70	2.62	2.55	2.47	2.39	2.30	2.22
192	14.00	0.50	1.00	1.50	2.00	2.50	3.00	3.50	4.00	4.50	5.00	5.50	6.00	6.50	7.00	7.50	8.00	8.50	9.00	9.50	10.00	10.50	11.00	11.50	12.00	12.50

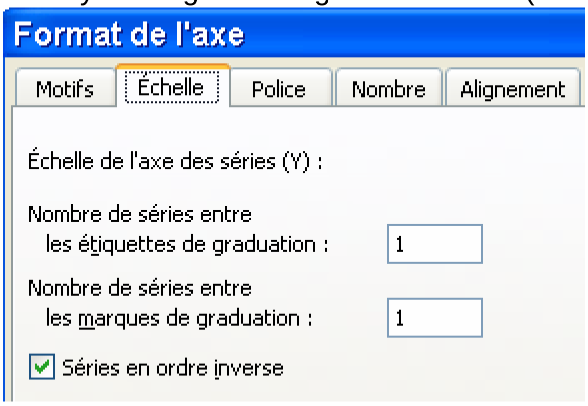
	AA	AB	AC	AD	AE	AF	AG	AH	AI	AJ	AK	AL	AM	AN	AO	AP	AQ	AR	AS	AT	AU	AV	AW	AX
164	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
165	0.10	0.10	0.10	0.10	0.10	0.10	0.09	0.09	0.09	0.09	0.08	0.08	0.08	0.08	0.07	0.07	0.07	0.07	0.07	0.07	0.07	0.07	0.07	0.07
166	0.21	0.21	0.21	0.20	0.20	0.19	0.19	0.18	0.18	0.17	0.17	0.16	0.16	0.15	0.15	0.14	0.14	0.14	0.14	0.13	0.13	0.13	0.13	0.13
167	0.32	0.31	0.31	0.31	0.30	0.29	0.28	0.27	0.27	0.26	0.25	0.24	0.23	0.23	0.22	0.22	0.21	0.21	0.20	0.20	0.20	0.20	0.20	0.20
168	0.42	0.42	0.42	0.41	0.40	0.39	0.38	0.37	0.35	0.34	0.33	0.32	0.31	0.30	0.29	0.29	0.28	0.28	0.27	0.27	0.26	0.26	0.26	0.26
169	0.54	0.53	0.52	0.51	0.50	0.49	0.47	0.46	0.44	0.43	0.41	0.40	0.39	0.38	0.37	0.36	0.35	0.34	0.34	0.33	0.33	0.33	0.33	0.33
170	0.65	0.65	0.64	0.62	0.61	0.59	0.57	0.55	0.53	0.51	0.50	0.48	0.46	0.45	0.44	0.43	0.42	0.41	0.40	0.40	0.39	0.39	0.39	0.39
171	0.77	0.76	0.75	0.73	0.71	0.69	0.67	0.64	0.62	0.60	0.58	0.56	0.54	0.52	0.51	0.49	0.48	0.47	0.47	0.46	0.45	0.45	0.45	0.45
172	0.90	0.89	0.87	0.85	0.82	0.79	0.76	0.73	0.71	0.68	0.66	0.63	0.61	0.59	0.58	0.56	0.55	0.54	0.53	0.52	0.51	0.51	0.51	0.51
173	1.03	1.02	1.00	0.96	0.93	0.89	0.86	0.82	0.79	0.76	0.73	0.71	0.68	0.66	0.64	0.63	0.61	0.60	0.59	0.58	0.57	0.57	0.57	0.57
174	1.19	1.16	1.13	1.08	1.04	1.00	0.95	0.91	0.87	0.84	0.81	0.78	0.75	0.73	0.71	0.69	0.67	0.66	0.65	0.64	0.63	0.63	0.62	0.62
175	1.36	1.32	1.27	1.21	1.15	1.10	1.04	1.00	0.95	0.92	0.88	0.85	0.82	0.79	0.77	0.75	0.73	0.71	0.70	0.69	0.68	0.68	0.68	0.68
176	1.57	1.49	1.41	1.33	1.26	1.19	1.13	1.08	1.03	0.99	0.95	0.91	0.88	0.85	0.83	0.80	0.78	0.77	0.75	0.74	0.74	0.73	0.73	0.73
177	1.86	1.67	1.55	1.44	1.36	1.28	1.22	1.16	1.10	1.06	1.01	0.98	0.94	0.91	0.88	0.86	0.84	0.82	0.81	0.79	0.79	0.78	0.78	0.78
178	1.98	1.80	1.65	1.54	1.45	1.36	1.29	1.23	1.17	1.12	1.07	1.03	1.00	0.96	0.93	0.91	0.89	0.87	0.85	0.84	0.83	0.83	0.82	0.82
179	2.04	1.87	1.74	1.62	1.52	1.44	1.36	1.29	1.23	1.18	1.13	1.09	1.05	1.01	0.98	0.96	0.93	0.91	0.90	0.89	0.88	0.87	0.87	0.87
180	2.07	1.92	1.80	1.68	1.59	1.50	1.42	1.35	1.29	1.23	1.18	1.14	1.10	1.06	1.03	1.00	0.98	0.96	0.94	0.93	0.92	0.91	0.91	0.91
181	2.09	1.96	1.84	1.73	1.64	1.55	1.47	1.40	1.34	1.28	1.23	1.18	1.14	1.10	1.07	1.04	1.02	0.99	0.98	0.96	0.95	0.95	0.94	0.94
182	2.10	1.98	1.88	1.77	1.68	1.59	1.52	1.45	1.38	1.32	1.27	1.22	1.18	1.14	1.11	1.08	1.05	1.03	1.01	1.00	0.99	0.98	0.98	0.98
183	2.11	2.00	1.90	1.80	1.71	1.63	1.55	1.48	1.42	1.36	1.31	1.26	1.21	1.18	1.14	1.11	1.08	1.06	1.04	1.03	1.02	1.01	1.01	1.01
184	2.12	2.02	1.92	1.83	1.74	1.66	1.58	1.51	1.45	1.39	1.34	1.29	1.24	1.20	1.17	1.14	1.11	1.09	1.07	1.05	1.04	1.04	1.03	1.03
185	2.12	2.03	1.94	1.85	1.76	1.68	1.61	1.54	1.48	1.42	1.36	1.31	1.27	1.23	1.19	1.16	1.14	1.11	1.09	1.08	1.07	1.06	1.05	1.05
186	2.13	2.04	1.95	1.86	1.78	1.70	1.63	1.56	1.50	1.44	1.39	1.34	1.29	1.25	1.22	1.18	1.16	1.13	1.11	1.10	1.09	1.08	1.07	1.07
187	2.13	2.04	1.96	1.87	1.79	1.72	1.65	1.58	1.52	1.46	1.40	1.35	1.31	1.27	1.23	1.20	1.17	1.15	1.13	1.11	1.10	1.09	1.09	1.09
188	2.13	2.05	1.96	1.88	1.80	1.73	1.66	1.59	1.53	1.47	1.42	1.37	1.32	1.28	1.24	1.21	1.18	1.16	1.14	1.12	1.11	1.10	1.10	1.10
189	2.13	2.05	1.97	1.89	1.81	1.74	1.67	1.60	1.54	1.48	1.42	1.37	1.33	1.29	1.25	1.22	1.19	1.17	1.15	1.13	1.12	1.11	1.11	1.11
190	2.13	2.05	1.97	1.89	1.81	1.74	1.67	1.60	1.54	1.48	1.43	1.38	1.33	1.29	1.26	1.22	1.20	1.17	1.15	1.14	1.12	1.12	1.11	1.11
191	2.13	2.05	1.97	1.89	1.81	1.74	1.67	1.60	1.54	1.48	1.43	1.38	1.33	1.29	1.26	1.22	1.20	1.17	1.15	1.14	1.12	1.12	1.11	1.11
192	13.00	13.50	14.00	14.50	15.00	15.50	16.00	16.50	17.00	17.50	18.00	18.50	19.00	19.50	20.00	20.50	21.00	21.50	22.00	22.50	23.00	23.50	24.00	24.50

Now, select the result area and choose the good graph options (Excel 2003) :

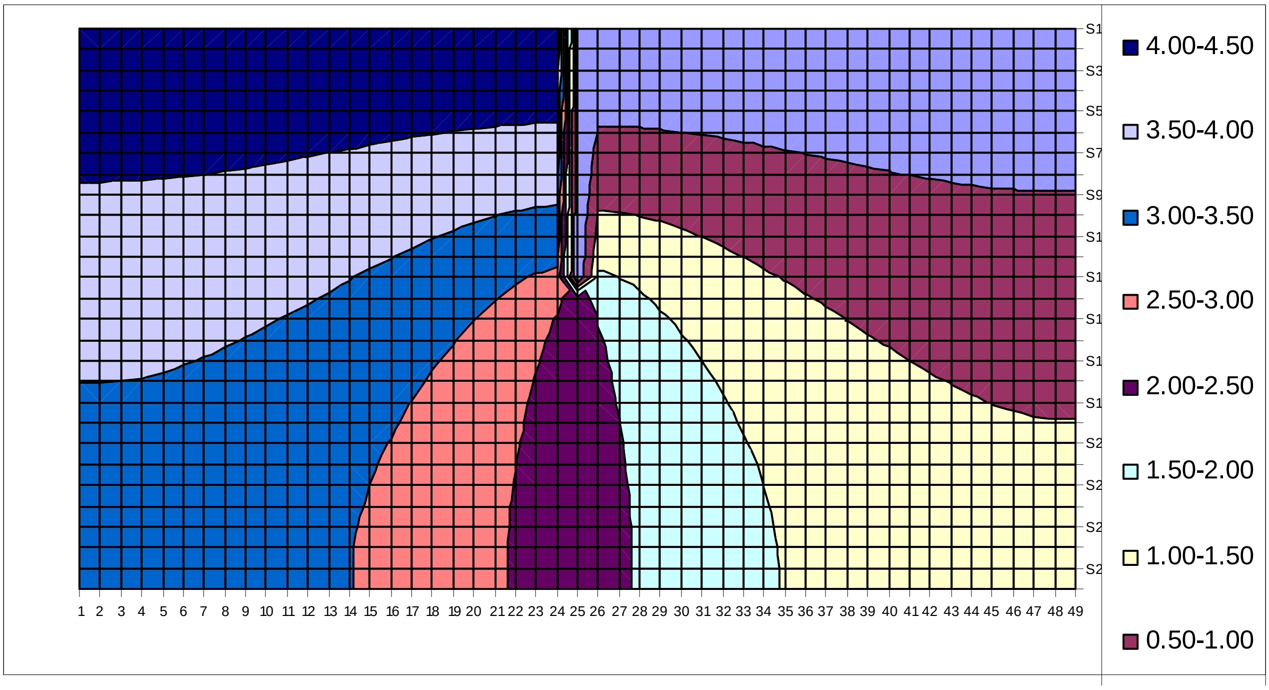


Then by clicking on the left vertical axis :

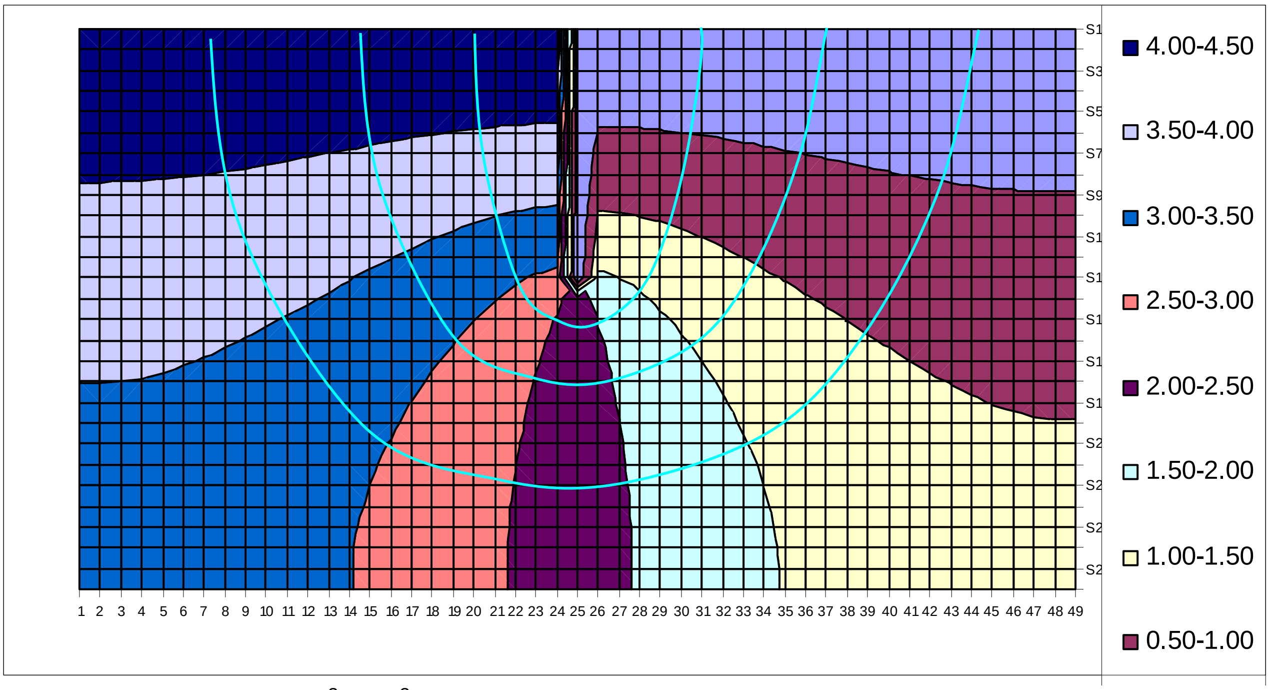
And by clicking on the right vertical axis (otherwise the graph appears upside down !):



Finally, one obtains the head graph :



Of course that macro only plots the head lines, as it is the head that follows the Laplace's equation. The flow lines must be drawn by hand, trying to draw "square" meshes.



$Q = k \cdot h \cdot N_f / N_d$ = 6. 10^{-3} x10^{-3}x4,5x 4/9 x 60x60x24= 1.04 m³/day / m of wall

10.6.4 Example – Stranded[5] Vessel Rescue

The figure 10 shows a long vessel, 20 metres wide, stranded on a sand bank. It is proposed to inject water into a well point, 10 metres down, under the centre of the vessel to assist in towing the vessel off. The water depth is 1 metre.

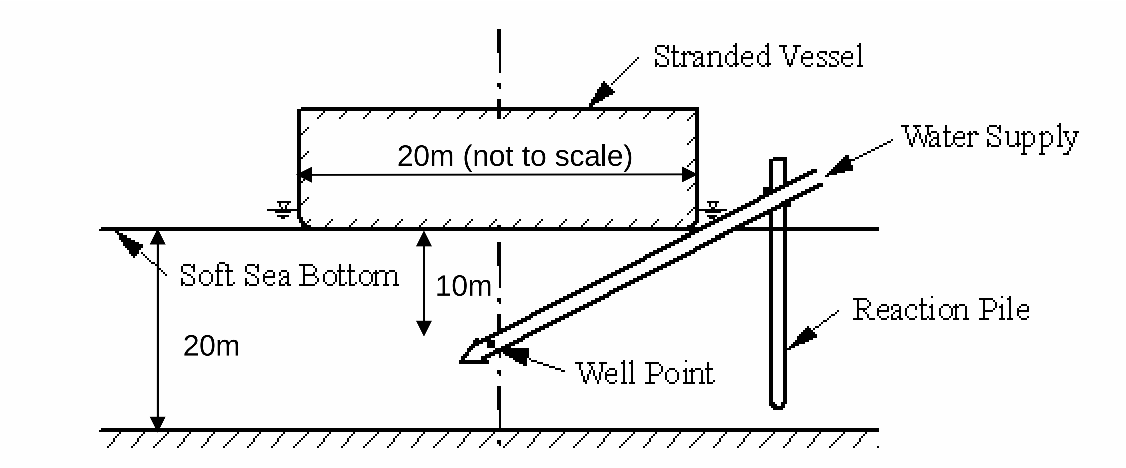


Figure 10 Stranded Vessel Example

The sand has a permeability of 3×10^{-4} m/sec. Assuming that a pressure head of 50 m can be applied at the well point :

1. draw the flow net by hand (and check eventually with the Excel macro) :

[5] échoué

2. calculate:
   The pore water pressure distribution across the base of the vessel
   The total upthrust due to this increase in pore water pressure
   The rate at which water must be pumped into the well point.

10.6.5 Exercise

2. The figure 11 shows a concrete dam lying on a permeable soil layer (k = 12.5 10^{-3} mm/s).
   a) draw the flow net
   b) calculate the total flow, per m width of dam, passing under the dam
   c) calculate the upthrust due to the water flow under the dam.

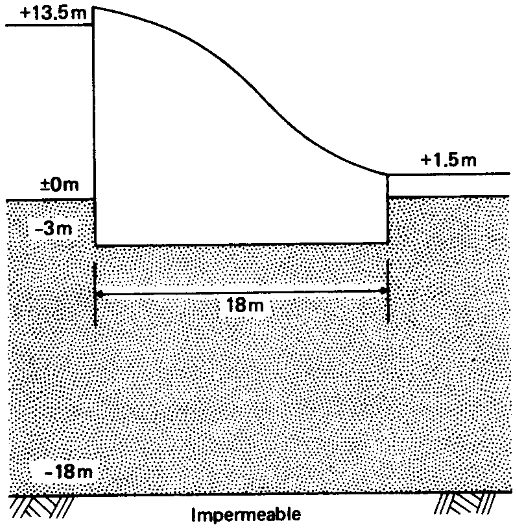


Figure 11

10.7 Flow Nets For Anisotropic Materials

10.7.1 Introduction

Many soils are formed in horizontal layers as a result of sedimentation through water. Because of seasonal variations such deposits tend to be horizontally layered and this results in different permeabilities in the horizontal and vertical directions.

10.7.2 Flow nets for soil with anisotropic permeability

It can be demonstrated that in the case of plane flow in an anisotropic material having a horizontal permeability k_H and a vertical permeability k_V, the solution can be reduced to that of flow in an isotropic material by doing a variable change

$$\bar{x} = \frac{x}{\alpha}$$

and

$$\bar{z} = z$$

with

$$\alpha = \sqrt{\frac{k_H}{k_V}}$$

So the flow in anisotropic soil can be analysed using the same methods (including sketching flow nets) that are used for analysing isotropic soils.

Example - Seepage in an anisotropic soil

Suppose we wish to calculate the flow under the dam shown in Figure 12;

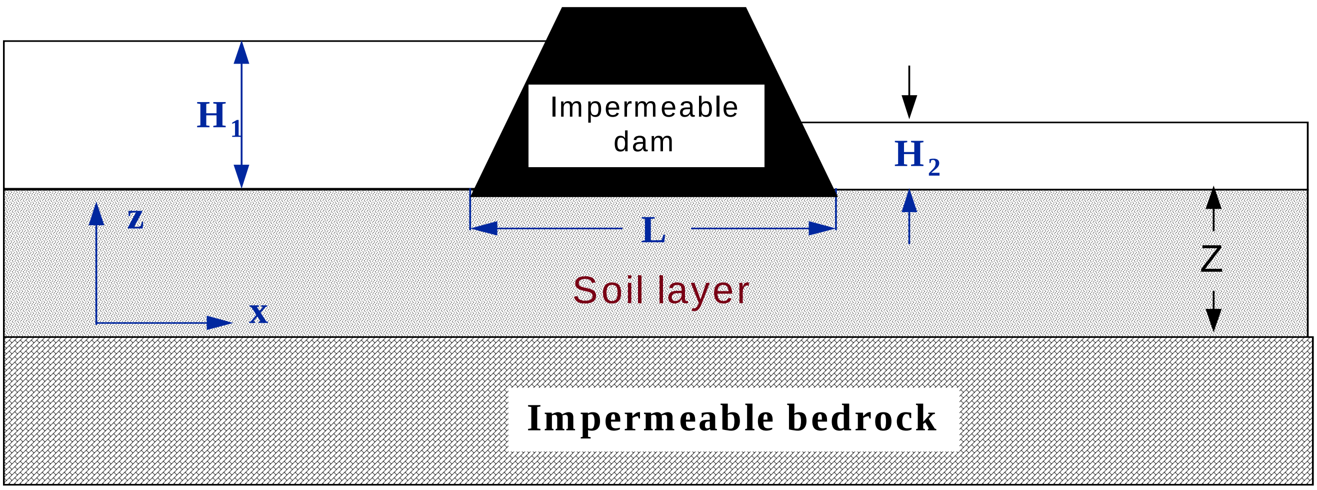


Fig. 12 Dam on a permeable soil layer over impermeable rock (natural scale)

For the soil shown in Fig. 12 it is found that $k_H = 4 \times k_V$ and therefore

$$\alpha = \sqrt{\frac{4 \times k_V}{k_V}} = 2$$

so

$$\bar{x} = \frac{x}{2}$$

$$\bar{z} = z$$

In terms of transformed co-ordinates this becomes as shown in Figure 13

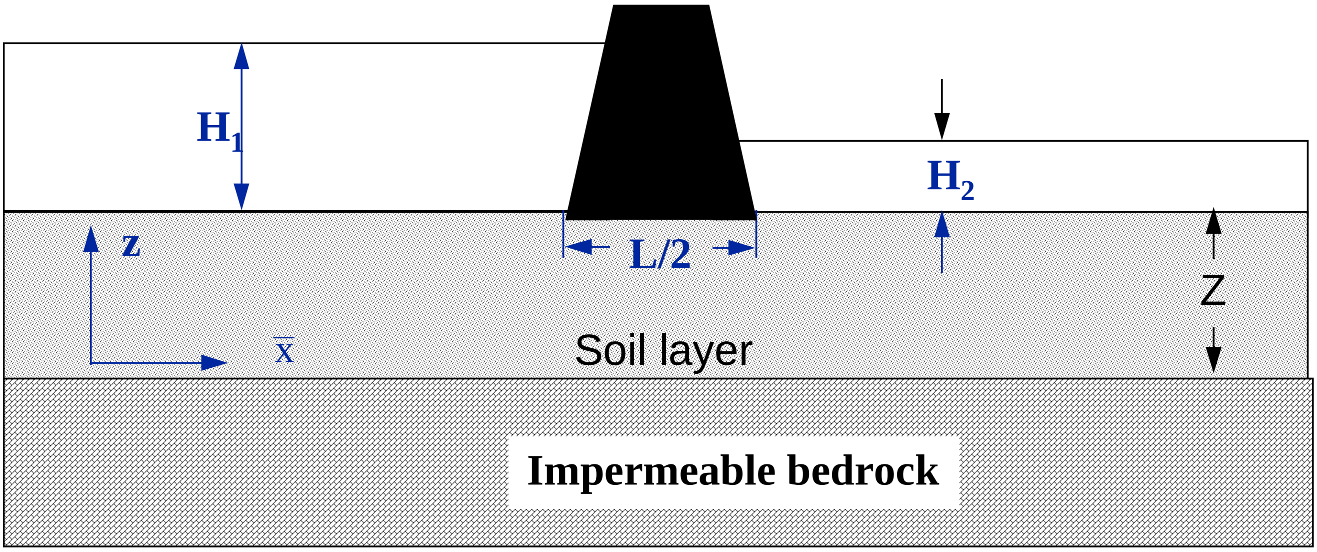


Fig. 13 Dam on a permeable layer over impermeable rock (transformed scale)

The flow net can now be drawn in the transformed co-ordinates and this is shown in Fig.14

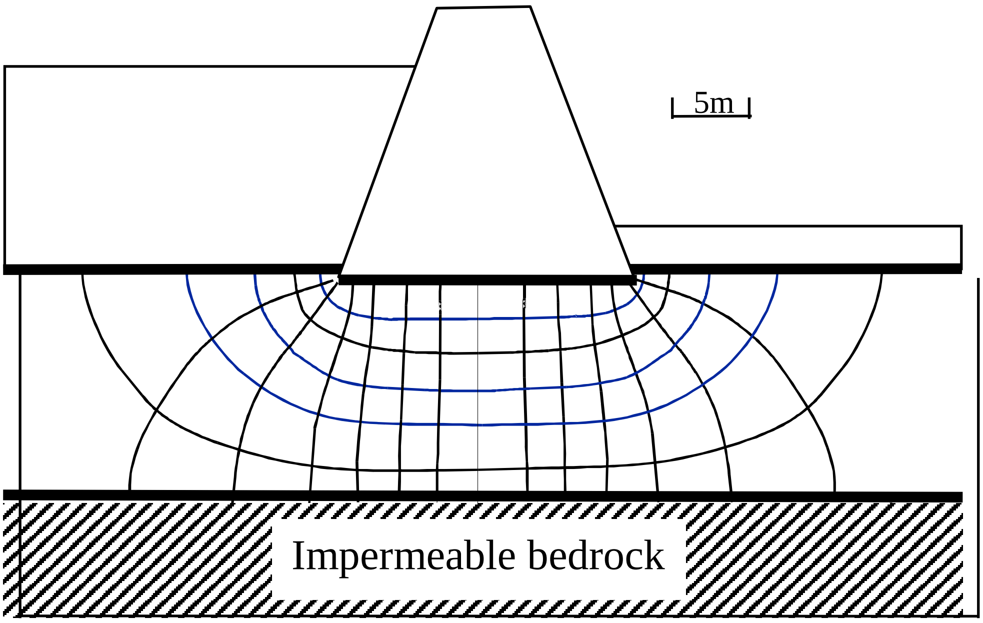


Fig. 14 Flow net for the transformed geometry

It is possible to use the flow net in the transformed space to calculate the flow underneath the dam by introducing an equivalent permeability

$$k_{eq} = \sqrt{k_H k_V}$$

A rigorous proof of this result will not be given here

Example

Suppose that in Figure 12 H_1 = 13m and H_2 = 2.5m, and that $k_V = 10^{-6}$ m/sec and $k_H = 4 \times 10^{-6}$ m/sec The equivalent permeability is:

$$k_{eq} = \sqrt{(4 \times 10^{-6}) \times (10^{-6})} \quad = \quad 2 \times 10^{-6} \text{ m / sec}$$

The total head drop is 10.5 m.
There are 14 head drops and thus Nd=14.
There are 6 flow tubes and thus Nf=6

The flow underneath the dam is, $\Delta Q = k_{eq} \, \Delta h \, N_f/N_d = (2\times10^{-6})\times10.5\text{x}6/14 = 9.0\times10^{-6}$ m^3/sec(/m width of dam)

For a dam with a width of 50 m, $Q = 450 \times 10^{-6}$ m^3/sec = 41.47 m^3/day

10.8 Effect of water flow in a soil mass

Introduction

Fig. 15 illustrates equilibrium conditions in a column of soil. The left-hand tank contains water and is connected to the right-hand tank containing soil and water. When the water level is the same in both tanks. there will be no flow of water through the soil.

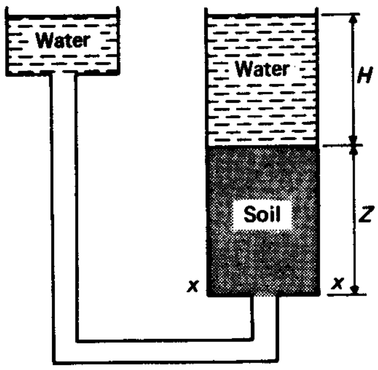


Figure 15

At level xx :

$\sigma = H \,.\, \gamma_w + Z \,.\, \gamma_{sat}$ (total stress)
$u = (H+Z) \,.\, \gamma_w$ (pore water pressure)
$\sigma' = Z \,.\, (\gamma_{sat} - \gamma_w)$ (contrainte effective = poids de la colonne de sol déjaugé)

Any change from equilibrium conditions will cause water to flow through the soil and this will alter the effective stress and pore water pressure. As it flows, the water exerts a frictional drag on the soil particles and the effect of this force is known as the seepage pressure.

(a) Flow downwards through the soil.
If the left-band tank is lowered, and the level of water in the right-hand tank is maintained, water will flow downwards through the soil (Fig. 16).

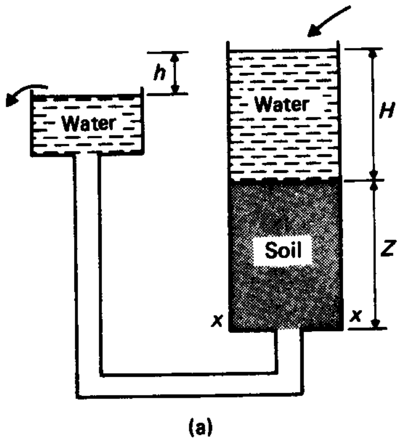


Figure 16

At level x-x

$\sigma = H \cdot \gamma_w + Z \cdot \gamma_{sat}$

$u = (H+Z-h) \cdot \gamma_w$

$\sigma' = Z \cdot (\gamma_{sat} - \gamma_w) + h \cdot \gamma_w$

Thus the effective pressure is increased by $h.\gamma_w$. This quantity $h.\gamma_w$ is the seepage pressure exerted by the flowing water.

(b) Flow upwards through the soil.

If the left-hand tank is raised, water will flow upwards through the soil (Fig. 17).

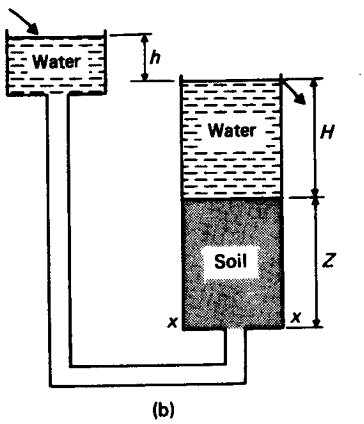


Figure 17

At level x-x

$\sigma = H \cdot \gamma_w + Z \cdot \gamma_{sat}$

$u = (H+Z+h) \cdot \gamma_w$

$\sigma' = Z \cdot (\gamma_{sat} - \gamma_w) - h \cdot \gamma_w$

Thus the effective pressure is decreased by $h.\gamma_w$, the amount of seepage pressure.

If h is increased, it may happen that the effective stress σ' vanishes : it is the heave due to seepage of water in the ground also called piping[6]. This kind of phenomenon may be catastrophic and will be developed in the next paragraph.

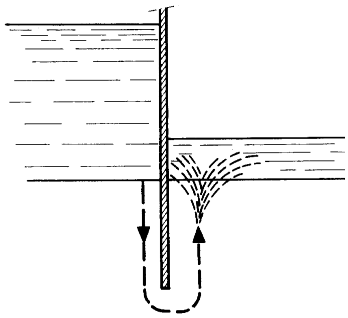


Figure 18 Piping

10.8.2 Seepage Force : general formulation

It can be demonstrated that the seepage force acting on a soil volume dV, is a force acting in the direction of the flow given by the formula:

$$\vec{J} = \gamma_w . \vec{i} . dV$$

Where :

$\vec{i}$ is the hydraulic gradient in the direction of the flow.

dV is a soil volume subjected to three forces :

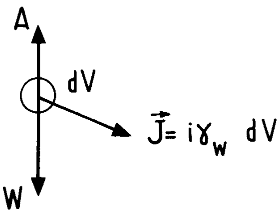


Figure 19 Seepage Force

- its own weight W = γ_{sat} . dV

- the buoyancy (Archimede's thrust) A = γ_w .dV

- the seepage force $\vec{J} = \vec{j}.dV = \gamma_w . \vec{i} . dV$

[6] « renard »

10.9 Hydraulic Heave or Piping

Many dams on soil foundations have failed because of the sudden formation of a piped shaped discharge channel. As the store water rushes out, the channel widens and catastrophic failure results. This results from erosion of fine particles due to water flow. Another situation where flow can cause failure is in producing 'quicksand' conditions. This is also often referred to as piping failure (Fig. 20).

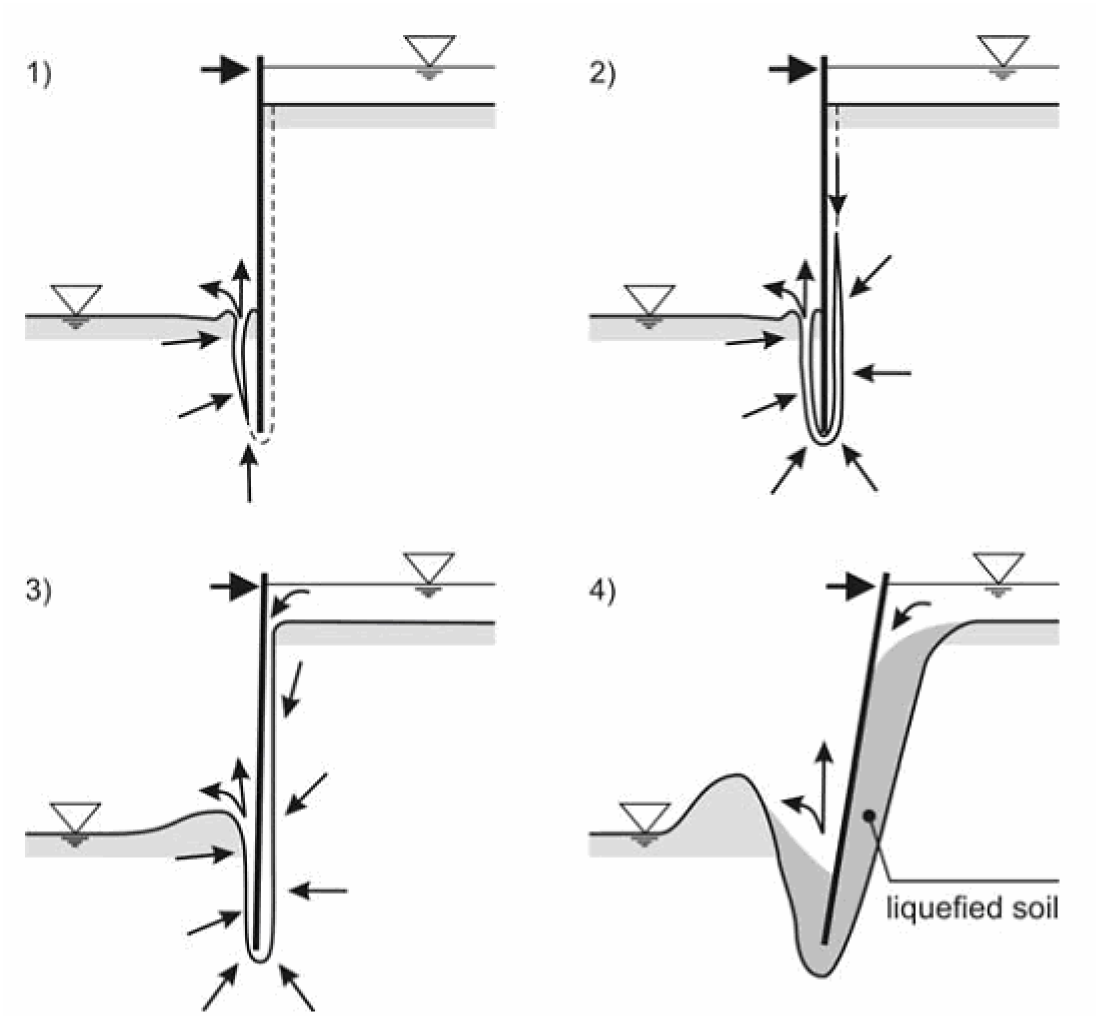
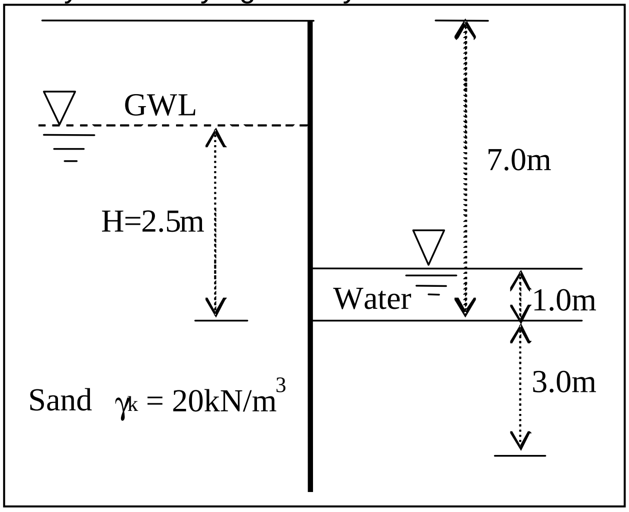


Figure 20 Piping Failure : 1) initiation and first deterioration, 2) regressive erosion, 3) formation of flow channel, 4) liquefaction and collapse

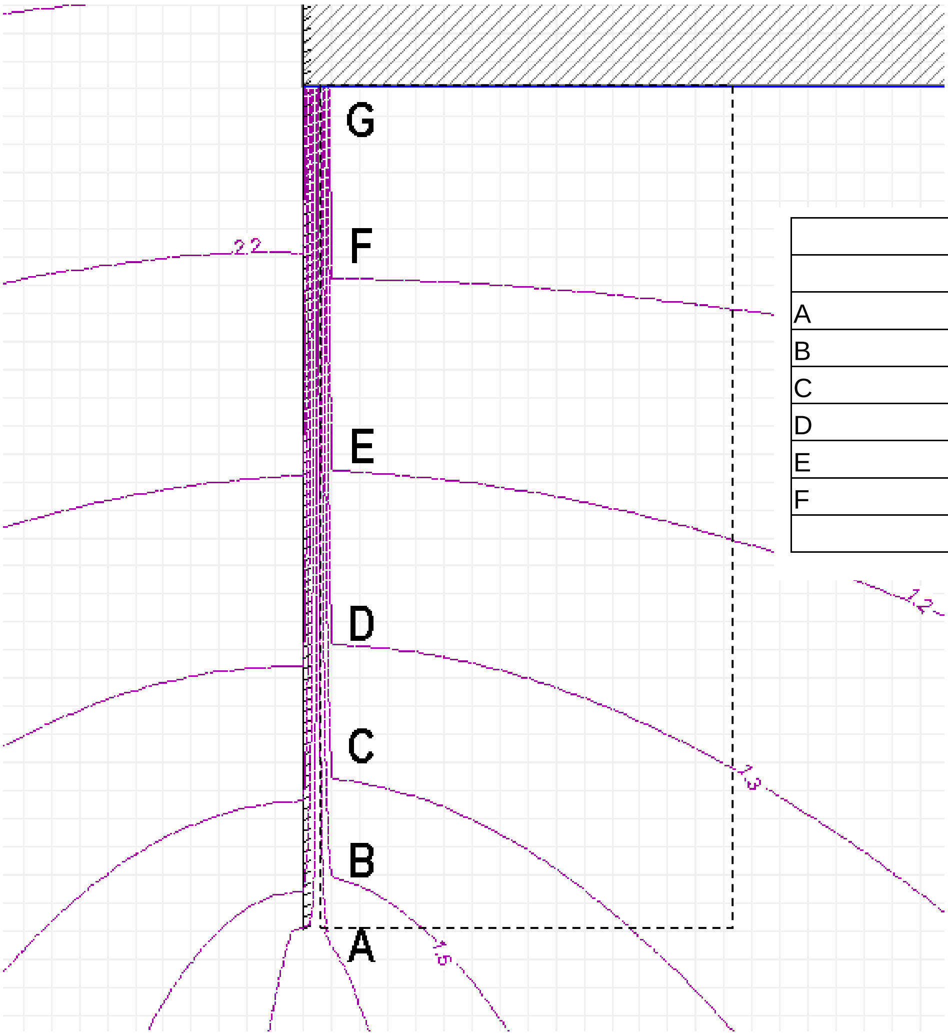
We will now show how to check the danger of piping in an example.

10.9.1 Exercise

Verify the safety against hydraulic heave in the following case.



The flow net, obtained with the Visual AEM freeware, gives the following equipotentials around the base of the wall. (The datum is at the ground level at the right of the wall.)



	h	z
	m	m
A	1.6	-3
B	1.5	-2.81
C	1.4	-2.46
D	1.3	-1.99
E	1.2	-1.36
F	1.1	-0.69
	1	0

As already mentioned in the chapter 8 about Eurocode 7, when considering a limit state of failure due to heave by seepage of water in the ground (HYD), it shall be verified, for every relevant soil column, that :

1. the design value of the destabilising total pore water pressure ($u_{dst;d}$) at the bottom of the column is less than or equal to the stabilising total vertical stress ($\sigma_{stb;d}$) at the bottom of the column : $u_{dst;d} \leq \sigma_{stb;d}$, or
2. the design value of the seepage force ($S_{dst;d}$) in the column is less than or equal to the submerged weight[7] ($G'_{dst;d}$) of the same column $S_{dst;d} \leq G'_{stb;d}$

The partial factor set for this case are:

Parameter		**Symbol**	**HYD - Partial factor set**
Permanent action (G)	**Unfavourable**	$\gamma_{G,\,dst}$	1.35
	Favourable	$\gamma_{G,\,stb}$	0.9
Variable action (Q)	**Unfavourable**	$\gamma_{Q,\,dst}$	1.5
	Favourable	-	0.0

z	u/γw	u=(h-z).γw.1.35	σtot=γsat*z*0.9+γw.1.35	u<σtot ?	i=dh/ds	Seepage Force per unit	
m	m	kN/m²	kN/m²			kN /m²	
-3	4.6	62.1	67.5	YES	0.526315789	-1.35	
-2.81	4.31	58.185	64.08	YES	0.285714286	-1.35	
-2.46	3.86	52.11	57.78	YES	0.212765957	-1.35	
-1.99	3.29	44.415	49.32	YES	0.158730159	-1.35	
-1.36	2.56	34.56	37.98	YES	0.149253731	-1.35	
-0.69	1.79	24.165	25.92	YES	0.144927536	-1.35	
0	1	13.5	13.5	YES			
				$S_{dst;d}$	Total Seepage Force (kN/m²)	-8.1	
				$G'_{stb;d}$	Total weight (kN/m²)	-13.5	

The wall is thus safe against piping, following Eurocode 7.

[7] Poids déjaugé